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Stochastic design improvement of an impact absorber

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Abstract

This work deals with a numerical investigation which has been carried out to study the crash behaviour of an automotive substructure located in the rear part of a vehicle, using a new stochastic methodology, Stochastic Design Improvement (SDI). Fundamentally, an iterative Monte Carlo simulation procedure was executed, whose trials consisted of deterministic numerical analyses based on explicit finite element methodologies.

The SDI procedure has been applied to reduce of a given amount the maximum displacement of a rigid barrier impacting against the rear substructure of a vehicle in a particular crash test.

The procedure allowed an efficient solution to the considered problem and the final results were validated through experimental tests.

1 Introduction

It is well known that numerical simulations are used to reduce the number of physical prototypes needed when designing complex structures, what is of the uttermost importance in the case of vehicles, where a significant reduction of the final project costs can be achieved by cutting down the number of crash tests. That leads to the need for a special methodology, complete with procedures and practical tools, such as to make possible to explore in detail each aspect of the involved physical phenomenon.

It must be pointed out, however, that numerical simulations carried out according to a deterministic approach cannot take into account the scattering of the parameters which randomly influence both the manufacturing process and the impact conditions, which, in turn, induce heavy effects on the results of such a highly non-linear phenomenon as crash.

Another case where a deterministic analysis can be usefully substituted with a stochastic analysis is when a target result cannot be achieved because of the scatter of the governing parameters; it is obvious for the designer to look for a robust design, such as to be influenced by the variation of parameters to a least extent.

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This work deals with a numerical investigation carried out in the study of the crash behaviour of an automotive substructure located in the rear part of a vehicle and which makes use of a new stochastic methodology, Stochastic Design Improvement (SDI), as implemented in the commercial code ST-ORM [3], which can be coupled with specialized FEM codes as LS-DYNA [2] for the deterministic structural part of the process [4].

The relevance of this methodology is that it allows the designer to find those values of the design variables which bring the output objective variables as nearest to a chosen target. Fundamentally it is based on an iterative Monte Carlo simulation procedure and therefore the substructure approach enables the designer to speed up the analysis of a large amount of cases and at the same time to generate enough data for an evaluation of the new methodology.

Moreover, if a test campaign is carried out first to validate numerical results of deterministic kind, a simpler demonstrator can be used, thus enabling the designers to further reduce the costs, taking into account a more limited number of design variables.

In the following, after briefly recalling the methodological aspects and the SDI technique, a case study is shown, where where the improvement process of the crash behaviour of a vehicle substructure is described [6, 10].

The used substructure was made in steel, but the adopted technique can be applied also to cases where lightweight materials are used and where the effects of the scatter of manufacturing parameters can be still more relevant.

2 Methodological aspects

If we consider the n-dimensional space, which is defined by the random variables which govern the problem in hand ("design variables") and which consist of geometrical, material, loads, environmental and human factors, we can observe that those sets of coordinates ($\mathbf{x}$) that determine the failure are located generally in a domain, which is thus defined as failure domain (Ω_f), in opposition to the whole rest of the same space that, corresponding to the safety condition, is known as the safety domain (Ω_s).

In the present context the concept of failure doesn't correspond to the case of physical damage, but it simply indicates the condition where the response of the structure doesn't match with the design requirements; therefore, both a fatigue life shorter than expected and a beam deflection larger than required define a "failure" case: it follows that in general the assigned requirement is met by many – even infinite – designs, each of them being characterized by a different probability of failure. The probabilistic analysis has the task to take as reference the design which, among all other cases, exhibits the largest probability to occur, thus defining the so called Most Probable Point (MPP) or Design Point (DP). A performance function is often introduced to indicate the extent by which the design is far from the failure case.

To simplify the following considerations, let us limit to the most common case where the performance value is assumed as zero (failure case) and that the limit requirements are fully prescribed. In the thus simplified hypothesis, the general problem of the probabilistic analysis of a structure is to evaluate, given the distributions of all the variables belonging to **x**, the probability that the response of the structure corresponds to a point whatever in the failure region, thus defining the probability of failure, P_f.

Therefore, if $f_x(x)$ is the joint probability density of all the said random variables **x**, such probability of failure is given by:

$$P_f = \int_{\Omega_f} f_x \cdot dx = \int_{\Omega_f} f_{x_1 x_2 \ldots x_n} \left(x_1 x_2 \ldots x_n \right) \cdot dx_1 dx_2 \ldots dx_n \tag{1}$$

Unfortunately such expression is just a formal one in the most cases, because the involved integral is of complex definition and primarily of impossible solution in a closed form, at most when dealing with evolutionary problems, where characteristics such as resistance and loads can vary with time; for such reasons, approximate methodologies of evaluation of the probability of failure have been introduced, which can be considered of two main typologies (with different approximations and with a lot of existing variations):

1. methodologies that use the limit state surface (LSF: the surface that constitute the boundary of the failure region) concept: they belong to a group of techniques that model variously the LSF in both shape and order and use such approximate surface to obtain an approximate probability of failure; among these, for instance, particularly used are the FORM (First Order Reliability Method) and the SORM (Second Order Reliability Method), that represent the LSF respectively through the hyper-plane tangent to the same LSF in the design point or through an hyper-paraboloid of rotation with the vertex in the same point.

2. Simulation methodologies, which are of particular importance in presence of complex problems: fundamentally they use the Monte-Carlo (MC) technique for the numerical evaluation of the specified integral and therefore they define the probability of failure on a frequencial approach [7, 8].

As it is known, the application of the MC method is based on the execution of a certain number, N, of experiments; for each one of those trials a set of design variables is extracted randomly and the resultant behaviour of the structure is analyzed, simply recording if it corresponds to the failure condition or not: if the failure condition is verified N_f times, the probability of failure is simply furnished frequencially by the ratio N_f/N.

Even if the MC method – which is now considered as founded on a sound base [9] – gives a result which is an unbiased estimator of the previous integral, obvious procedural difficulties arise, since only a finite number of experiments can be carried out, and therefore the obtained result is affected in many cases from a large interval of confidence or, alternatively, from a small level of confidence.

Consider, for instance, an hypothetical case for which the theoretical probability of failure is 10^{-3}; if 1.000 experiments are performed, it can happen that no trial gives a failure result, which would force to accept the impossible result of a probability of failure equal to zero; but one could also obtain that, because of the random extraction of the variables, the failure condition is verified 2, 3 or also 5 times, from which the evaluated probability of failure would exceed the real one many times.

The most common case is the first one, so we can say that with a few number of experiments the calculated probability of failure is in general (but not theoretically) smaller than the real one; this happens because in the random extraction the design variables are mostly positioned approximately around their mode and, if the project has been realized correctly from a deterministic standpoint, such values necessarily correspond to a condition of safety: therefore, the largest part of the experiments will tend to exclude the condition of failure unless a large number of trials is carried out.

In practical problems the previous danger is very relevant, as one is interested to evaluate very small probabilities of failure, at present down to 10^{-9} if many human lives are at a stake; in any case, even if the knowledge of the order of magnitude of the probability of failure can often be considered as sufficient, it is clear that a huge number of experiments have to be carried out, which implies very time-expensive procedures, expecially if the results are obtained by using a numerical code on a large structure.

To obviate to such serious problem, modifications of the MC method have been introduced, which are commonly known as techniques for the reduction, or the optimization, of the coefficient of variation (of the result); among these particularly interesting is the so-called method of the Response Surface (RSM), with which an approximate model of the LSF is built on the base of the results of a first set of experiments and then progressively improved with the following trials, in order to obtain a numerical form of the surface such as to let the user to adopt procedures similar to those used in the previous group of techniques. Other procedures which are increasingly adopted today include importance sampling, directional simulation, domain restricted sampling and others.

All such techniques are especially useful in structural field, where the LSF is usually impossible to define analitically, since they allow to reduce the number of the MC experiments, that are very expensive from a computational point of view for a complex structure or a particular phenomenon, such as the impact of a vehicle.

Obviously, the LSF obtained with a few number of experiments is approximate, so an alternative way to proceed is to continue with the MC method and with new structural analyses observing the subsequent changes of the LSF and arresting only when it doesn't change anymore.

The aim of the previous discussion was the evaluation of the probability of failure of a given structure, with assigned statistics of all the design variables involved; the reciprocal problem can be considered now, that is the definition of the design which is characterized by a given probability of failure, i.e. of mismatch with the prescribed requirements.

Usually, that problem can be solved for simple cases by assigning the coefficients of variation of the design variables and looking for the corresponding mean values; the above mentioned hypothesis referring to the constancy of the coefficients of variation is usually justified with the connection between variance and quality levels of the production equipments, not to mention the effect of the nowadays probabilistic techniques, which let introduce just one unknown in correspondence of each variable.

In any case, one must consider that, at least in the structural field for the case of large deformations, the relationship between the statistic of the response and that of a generic design variable for a complex structure is in general non-linear; it is in fact evident from fig. 1 that two different mean values for the random variable x, say x_A and x_B, even in presence of the same standard variation, correspond to responses centered in y_A and y_B, whose coefficients of variation are certainly very different from each other.

It follows from what above that the statistics of the result can be largely dependent not only on the type of the statistics of the design variables, but also from the starting point of the analysis, i.e. from the initial mean value of the same variables; it is therefore possible to look for those values of the same design variables which correspond to a result, i.e. a design, which is characterized by the smallest variance, and that is, in fact, the definition of a "robust design" as given by Taguchi, in a field which is not strictly connected with the evaluation of a probability of failure.

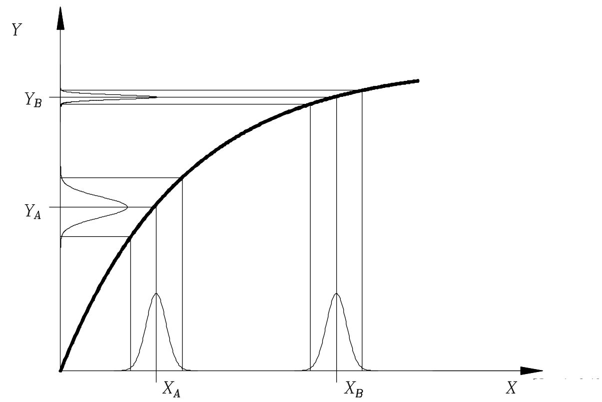

Figure 1: non linear relationship between input and output variable.

As it is nowadays accepted, however, the requirements of the amplitude of the variance of the result can be substituted with the need to satisfy an assigned condition whatever and, among others, a given probability of failure.

Consequently, while in the usual probabilistic problem we are looking for the consequences on the realization of a product arising from the assumption of certain distributions of the design variables, in the theory of the robust design the procedure is reversed, now looking for those statistical parameters of the design variables such as to produce an assigned result (target), as characterized by a given probability of failure.

It must be considered, however, that no hypothesis can be introduced about the uniqueness of the result, in the sense that more than one design can exist such as to satisfy the assigned probability, and that the result depends on the starting point of the analysis, which is a well known problem also in other cases of probabilistic analysis. Therefore, the most useful way to proceed is to define the target as a function of a given design solution, for example of the result of a deterministic procedure, in order to obtain a feasible or convenient solution, as it will be shown in the next sections.

3 The SDI process

A set of techniques has been introduced in order to obtain a robust design, i.e. one whose behaviour is rather insensitive to all variations of the main variables, or, what is the same, a design whose statistics are characterized by the smallest standard deviation, as a function of the statistics of input; as we reported above, the same procedure can be used to obtain a design which exhibits an assigned probability of failure, by means of a correct choice of the mean values of the design variables.

This problem can be effectively dealt with by an SDI (Stochastic Design Improvement) [5] process, which is carried out by means of several MC series of trials (runs) as well as of the analysis of the intermediate results.

In fact, input – i.e. design variables $\mathbf{x}$ – and output – i.e. target $\mathbf{y}$ – of a mechanical system can be connected by means of a functional relation of the type

$$y = F(x) \quad , \tag{2}$$

which in the largest part of the applications cannot be defined analytically, but only ideally deduced because of the its complex nature; in practice, it can be obtained by considering a sample $\mathbf{x}_i$ and examining the response $\mathbf{y}_i$, which can be carried out by a simulation procedure and, first of all, by one of M-C techniques, as recalled above. Considering a whole set of M-C samples, the output can be expressed by a linearized Taylor expansion centered about the mean values of the control variables, as

$$y_i = F(\mu_x) + \frac{dF}{dx}(x_i - \mu_x) = \mu_y + G(x_i - \mu_x) \quad , \tag{3}$$

where μ_i represents the vector of mean values of input/output variables and where the gradient matrix $\mathbf{G}$ can be obtained numerically, carrying out a multivariate regression of $\mathbf{y}$ on the $\mathbf{x}$ sets obtained by M-C sampling. If $\mathbf{y}_0$ is the required target, we could find the new $\mathbf{x}_0$ values inverting the relation above, i.e. by

$$x_0 = \mu_x + G^{-1}(y_0 - \mu_y); \tag{4}$$

as we are dealing with probabilities, the real target is the mean value of the output, which we compare with the mean value of the input, and, considering that, as we shall illustrate below, the procedure will evolve by an iterative procedure, it can be stated that the relation above has to be modified as follows, considering the update between the k-th and the (k+1)-th step:

$$\mu_{x0} = \mu_{x,k+1} = \mu_{x,k} + G^{-1}\left(\mu_{y,k+1} - \mu_{y,k}\right) = \mu_{x,k} + G^{-1}\left(\mu_{y0} - \mu_{y,k}\right). \quad (5)$$

The SDI technique is based on the assumption that the cloud of points corresponding to the results obtained from a set of MC trials can be moved toward a desired position in the N-dimensional space such as to give the desired result (target) and that the amplitude of the required displacement can be forecast through a close analysis of the points which are in the same cloud (fig 2): in effects, it is assumed that the shape and size of the cloud don't change greatly if the displacement is small enough; it is therefore immediate to realize that an SDI process is composed by several sets of MC trials (runs) with intermediate estimates of the required displacement.

It is also clear that the assumption about the invariance of the cloud can be maintaned just in order to carry out the multivariate regression which is needed to perform a new step – i.e. the evaluation of the **G** matrix – but that subsequently a new and correct evaluation of the cloud is needed; in order to save time, the same evaluation can be carried out every k steps, but of course, as k increases, the step amplituded has to be correspondently decreased.

It is also immediate that the displacement is obtained by changing the statistics of the design variables and in particular by changing their mean (nominal) values, as in the now available version of the method all distributions are assumed to be uniform, in order to avoid the gathering of results around the mode value. It is also to point out that sometimes the process fails to accomplish its task because of the existing physical limits, but in any case SDI allows to quickly appreciate the feasibility of a specific design, therefore making easier its improvement.

Of course, it may happen that other stochastic variables are present in the problem (the so called background variables): they can be characterized by any type of statistical distribution included in the code library, but they are not modified during the process.

Therefore, the SDI process is quite different for example from the classical design optimization, where the designer tries to minimize a given objective function with no previous knowledge of the minimum value, at least in the step of the problem formulation. On the contrary, in the case of the SDI process it is first stated what is the value that the objective function has to reach, i.e. its target value, according to a particular criterion which can be expressed in terms of maximum displacement, maximum stress, or other. The SDI process gives information about the possibility to reach the objective within the physical limits of the problem and determines which values the project variables must have in order to get it. In other words, the designer specifies the value that an assigned output variable has to reach and the SDI process determines those values of the project variables which ensure that the objective variable becomes equal to the target in the mean sense. Therefore, according to the requirements of the problem, the user defines a set of variables as control variables, which are then characterized from a uniform statistical distribution (natural variability) within which the procedure can let them vary, observing the corresponding physical (engineering) limits. In the case of a single output variable, the procedure evaluates the Euclidean or Mahalanobis distance of the objective variable from the target after each trial:

$$d_i = |y_i - y^*| \qquad i = 1, 2, \ldots, N, \tag{6}$$

where y_i is the value of the objective variable obtained from the i-th iteration, y^* is the target value and N is the number of trials per run. Then, it is possible to find among the worked trials that one for which the said distance gets the smallest value and subsequently the procedure redefines each project variable according to a new uniform distribution with a mean value equal to that used in such "best" trial. The limits of natural variability are accordingly moved of the same quantity of the mean in such way as to save the amplitude of the physical variability.

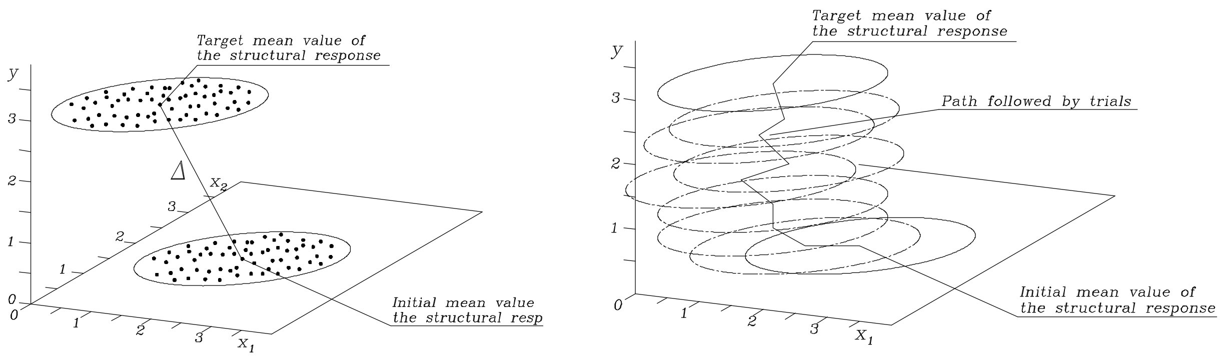

Figure 2: SDI process

If the target is defined by a set of output variables, the displacement toward the condition where each one has a desired (target) value is carried out considering the distance as expressed by:

$$d_i = \sqrt{\sum\nolimits_k \left(y_{i,k} - y_k^*\right)^2}, \tag{7}$$

where k represents the generic output variable. If the variables are dimensionally different it is advisable to use a normalized expression of the Euclidean distance:

$$d_i = \sqrt{\sum \omega_k \left(\delta_{i,k}\right)^2}, \tag{8}$$

where:

$$\begin{array}{lll} \delta_{i,k} = \frac{y_{i,k}}{y_k^*} - 1, & if & y_k^* \neq 0 \\ \delta_{i,k} = y_{i,k} & if & y_k^* = 0 \end{array}, \tag{9}$$

but in this case it is of course essential to assign weight factors ω_k to define the relative importance of each variable.

Several variations of the basic procedures are available; for example, it is possible to define the target by means of a function which implies an equality or an inequality, too; in the latter case the distance is to be considered null if the inequality is satisfied.

Once the project variables have been redefined a new run is performed and the process restarts up to the completion of the assigned number of shots. It is possible to plan a criterion of arrest in such way as to make the analysis stop when the distance from the target reaches a given value. In the most cases, it is desirable to control the state of the analysis with a real-time monitoring with the purpose to realize if a satisfactory condition has been obtained.

4 Approaching a test case

In the present work, the SDI procedure is applied to reduce the maximum value of the displacement in time of a rigid barrier that impacts the rear substructure of a vehicle (fig. 3 and fig. 4) in a crash test. The reasons which lie behind such a choice are to be found in the increasing interest in numerical analysis of crashworthiness of vehicles because of the more strict regulations concerning the protection of the occupants and related fields.

In Europe the present standards to be used in homologation of vehicles are more or less derived by U.S. Code of Federal Regulations, CFR-49.571, but ever increasing applications are done with reference to other standards, and first of all to EURONCAP. The use of such standards, whose are mainly directed to limit biomechanical consequences of the impact – which are controlled by referring the results to standard indexes related to different parts of human body – implies that, besides the introduction of specific safety appliances, as safety belts and airbags, the main strength of car body has to be located in the cell which hosts passengers, in order to obtain a sufficient survival volume for the occupants; the other parts of the vehicle are only subsidiary ones, because of the presence of absorbers which reduce the impact energy which is released on the cell.

Therefore, such considerations can induce designers to disregard the strength of those other parts, with the consequence that even light impacts can induce heavy and expensive damages on the car body. To limit those consequences, in recent times a new type of analysis has been introduced, which refers to low speed impacts and which is often supported by insurance firms – from which it is currently known as “insurance impact” – where the impact speed is limited to 16 km/h, while in the other tests the same impact speed is 56 or 65 km/h according to the specific standards.

Because of the small amount of energy released in such impacts, crash tests can be limited to substructures or small parts of the vehicle, which requires small apparatuses for experimental tests, or corresponding relatively small models in the case of numerical analyses. As the main scope of the present work was to test the capabilities of SDI technique, which, as it will be shown later, requires a large number of numerical tests, an obvious choice was to refer to such an impact and to limit our attention to a specific substructure located in the rear part of a

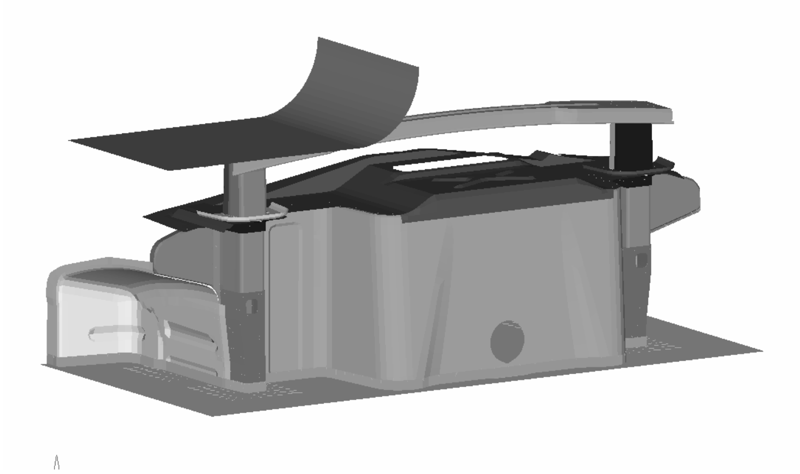

Figure 3: Global FEM model.

vehicle, with the consequence that rather fast runs were obtained even in presence of the fine mesh adopted, as it is always needed when simulating impact conditions.

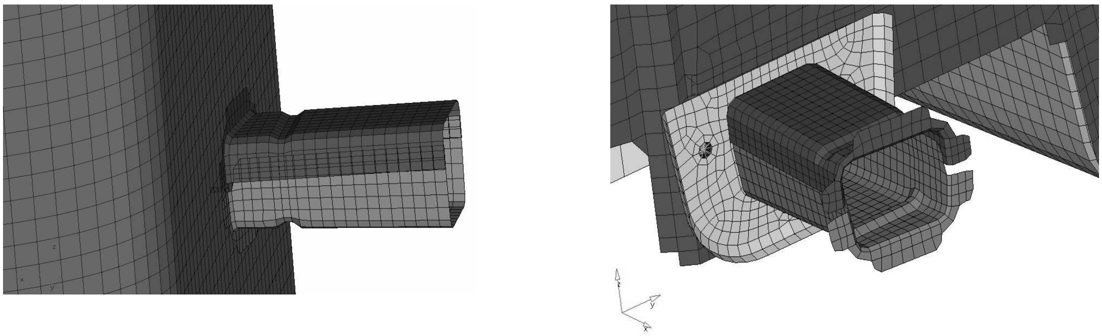

Figure 4: FEM model details (impact absorber).

5 Performing SDI analysis

We can add to all previous considerations that the present case study has been adopted as it is well known that vehicle components come from mass production, where geometrical imperfections are to be expected as well as a certain scatter of the mechanical properties of the used materials; then, it can't be avoided that the said variations induce some differences of response among otherwise similar components, what can be relevant in particular cases and first of all in impact conditions; the analysis which has been carried out was directed to define, through the use of the previously sketched procedure, the general criteria and the methodology required to develop a robust design of those vehicle components which are directly used to limit plastic deformations in impact (impact absorber). In our particular case, the study was carried out with reference to the mentioned substructure, whose effective behaviour in impact (hammer) conditions is known and is associated to those deterministic nominal values of the design variable actually in use, with the immediate objective to obtain a reduction of the longitudinal deformation of the impact absorber.

The substructure is a part of a rear frame of a vehicle, complete with cross-bar and girders, where impact absorbers are inserted; the group is acted upon by a hammer which is constrained to move in the longitudinal direction of the vehicle with an initial impact speed of 16 km/h; the FE model used for the structure consists of about 23400 nodes and about 21900 elements of beam, shell and contact type, while the hammer is modelled as a "rigid wall".

The thicknesses of the internal (lower in the fig. 3) and external (upper in the fig. 3) C-shaped plates of the impact absorbers have been selected as project variables, with a uniform statistical distribution in the interval [1.7mm÷1.9mm]; lower and upper engineering limits are respectively 1.5 mm and 2.1 mm.

This choice has been carried out by preliminary performing, by using the probabilistic module of ANSYS ver 8.0 [1] linked to the explicit FE module of LS-Dyna included in the same code, a sensitivity analysis of an opportune set of design variables on the objective variable, that is, as already stated before, the maximum displacement of the hammer.

The considered design variables involved in the sensitivity analysis have been chosen, besides the inner and outer thicknesses of the C-shaped profile of the impact absorbers, to be the mechanical properties of the three materials constituting the main components of the substructure (the unique young modulus and the three yielding stresses). In the following fig. 5 it is possible to appreciate the "relationship" existing between the chosen design variables and the objective variable by considering the scatter plots between them.

From these scatter plots it is clear that while the relationship existing between the thicknesses of the inner and outer C-shaped profile of the impact absorber and the objective variable is quite linear, as well as the relationship between the yielding stress of the impact absorber material and the same objective variable, a relationship between the other considered variables and the objective variable is undetermined.

In the following fig. 6, the results from the sensitivity analysis are also reported: it is quite evident that the only design variables which influence the objective one are the mechanical properties of the material of the impact absorber and the thicknesses of their profile.

A preliminary deterministic run, carried out with the actual design data of the structure gave for the objective variable a 95.94 mm "nominal" value, which was reached after a time of 38.6 ms from the beginning of the impact. The purpose of SDI in our case has been assumed to reduce this displacement by 10% with respect to this nominal value and therefore an 86.35 mm target value has been assigned.

The mechanical properties of the three materials constituting the absorbers and the rear

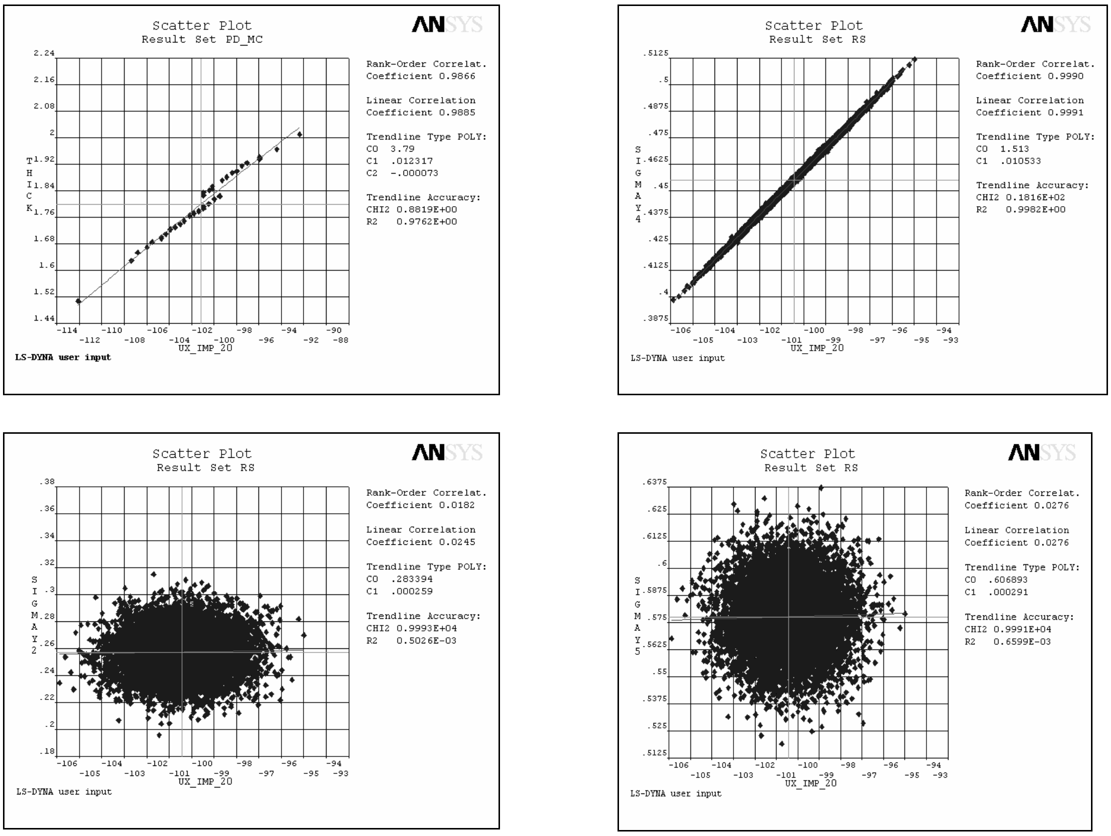

Figure 5: scatter plots of design variables vs. objective variable.

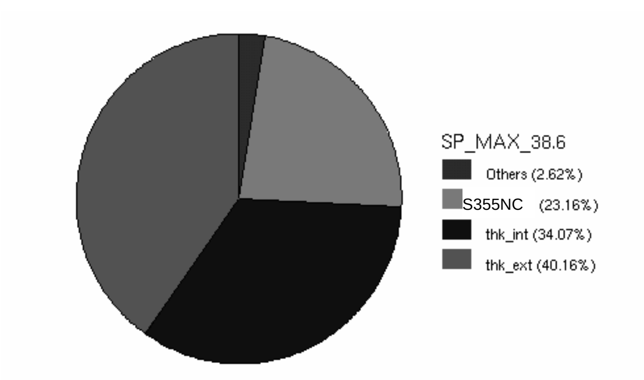

Figure 6: sensitivity analyses

crossbar of the considered substructure have also been considered as random; it has been assumed that their characteristic stress-strain curves may vary according to a uniform law within ±5% of the nominal value. This has been possible by introducing a scale factor for the characteristic curves of the materials, which has been considered as uniformly distributed in the interval [0.95, 1.05].

Moreover, four stress-strain curves have been considered for each material, corresponding to as many specific values of the strain-rate. The relationship among those curves and the static one has been represented, according to the method used in Ls-Dyna, by means of a scale factor which let us pass from one curve to another as a function of the strain-rate; also those factors have been assumed to be dependent on that applied to the static curve, in order to avoid possible overlapping.

Therefore, the simulation has involved 14 random variables, among which only 2 are considered as project variables; in the following Tab. 1 the properties of all the variables are listed.

Table 1: Properties of variables.

Variable	**Type**	**Distribution**	**Natural variability**	**Physical limits**
Internal plate thick.	Design var.	uniform	1.7-1.9	1.5-2.1
External plate thick.	Design var.	uniform	1.7-1.9	1.5-2.1
material scale factor DC04 strain-rate 0	independent var.	uniform	0.95-1.05	
material scale factor DC04 strain-rate 0.005	dependent var.			
material scale factor DC04 strain-rate 0.05	dependent var.			
material scale factor DC04 strain-rate 0.5	dependent var.			
material scale factor DP600 strain-rate 0	independent var.	uniform	0.95-1.05	
material scale factor DP600 strain-rate 0.005	dependent var.			
material scale factor DP600 strain-rate 0.05	dependent var.			
material scale factor DP600 strain-rate 0.5	dependent var.			
material scale factor S355NC strain-rate 0	independent var.	uniform	0.95-1.05	
material scale factor S355NC strain-rate 0.03	dependent var.			
material scale factor S355NC strain-rate 0.5	dependent var.			

As we said above, St-Orm used Ls-Dyna explicit solver for each deterministic FEM analysis and ran on a 2600 MHz bi-processor PC, equipped with a 2 Gb RAM; SDI processing required 9 runs with 25 shots each, with a total of 225 MC trials, and the time required to complete a single LS-Dyna simulation being of about 2 hours.

6 Analysis of results and conclusions

As we already pointed out in the previous paragraphs, the stochastic procedure develops through an MC iterative process where the input variables are redefined in each trial in such a way as to move the results toward an assigned target; therefore, we need first to assess what we mean as an attained target.

After every run the statistics of the output variables can be obtained, as well as the number of times that the target is reached, what can be considered as the probability of attainment of the target for the particular design, expressed through the mean values of the input variables.

It is noteworthy to specify that these data are only indicative, because the MC procedure developed within a single set of trials is not able to give convergent results due to the low number of iterations.

Therefore, considering the procedure as ended when a run is obtained where all trials give the desired target value, it is opportune to perform a final MC convergent process to evaluate the extent by which the target has been indeed reached, using the statistical distributions of the variables of the last run. For the same reason, a real-time monitoring can induce the designer to stop the procedure even if not all trials – but "almost" all - of the same run give the target as reached, also to comply with production standard and procedures.

Table 2: detailed data for every run.

	Thickness of the internal sheet [mm]			**Thickness of the external sheet [mm]**			**Displacement [mm]**		**Distance from target**	
run	left bound	mean value	right bound	left bound	mean value	right bound	mean value	std	mean value	std
1	1.7000	1.8000	1.9000	1.7000	1.8000	1.9000	95.9524	2.4504	0.0585	0.0270
2	1.7903	1.8903	1.9903	1.7909	1.8909	1.9909	91.5568	2.3917	0.0158	0.0190
3	1.7127	1.8127	1.9127	1.8322	1.9322	2.0322	92.5833	2.4903	0.0240	0.0232
4	1.7297	1.8297	1.9297	1.8795	1.9795	2.0795	91.0362	2.3111	0.0132	0.0144
5	1.7624	1.8624	1.9624	1.9475	2.0238	2.1000	88.8615	2.1464	0.0026	0.0061
6	1.7632	1.8632	1.9632	2.0000	2.0500	2.1000	88.0821	1.6937	0.0005	0.0016
7	1.7446	1.8446	1.9447	1.9259	2.01295	2.1	89.6456	2.2662	0.005384	0.01072
8	1.7831	1.8831	1.9831	1.9385	2.0193	2.1	88.5974	2.1700	0.002176	0.00664
9	1.8778	1.9778	2.0778	**1.8846**	**1.9846**	**2.0846**	87.6936	2.1465	0.0009655	0.00367

For what concerns our case study, the detailed data for every run are recorded in Tab. 2; if compared with the first run, the mean of the displacement distribution in the 9th run is reduced of 8.6% and 23/25 shots respect the target: therefore, the results of the 9th run may be considered as acceptable.

In the plots of fig. 7 and fig. 8 the values of the thickness of the internal and external plates of the impact absorber versus the current number of shots have been illustrated. The variable which was subjected to the largest modifications in the SDI procedure is the thickness of the external plate of the impact absorber and in fact from an analysis of sensitivity (fig. 6) it results to influence at the largest extent the distance from the target.

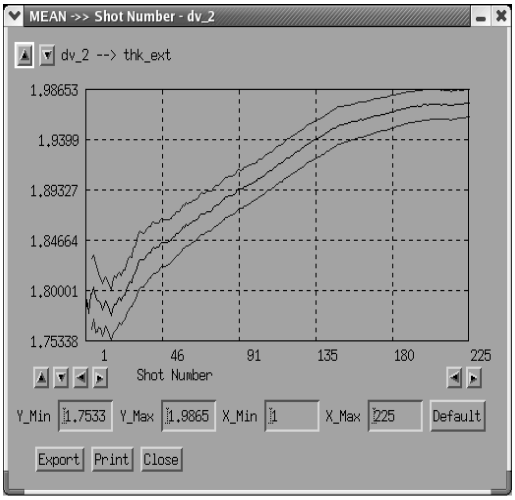

Figure 7: design variable vs. shot number.

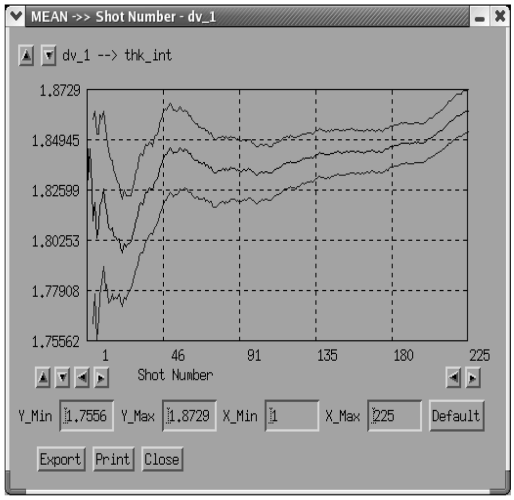

Figure 8: design variable vs. shot number.

For what concerns the other random variables, it results from the same analysis of sensitivity that only the material of the absorbers (S355NC in fig. 6) influences in a relevant measure the behaviour of the substructure.

In the plots of fig. 9 and fig. 10 the path followed by the mean value of the distribution of the displacement of the rigid barrier (the objective variable) and by its distance from the target versus the current number of iterations are illustrated; it is necessary to clarify that in our plots the relative distance to target is recorded.

It is possible to appreciate from the scatter plots of fig. 11 and fig. 12 how the output variable approaches the target value: in the 9th run, only 2 points fall under the red line that represents the target and in both cases the distance is less than 0.02 and that is why the 9th run has been considered a good one, in order to save more iterations.

In fig. 13 the experimental data related to the displacement of the rigid barrier vs. the time are recorded together with the numerical results obtained before and after the application of the SDI procedure.

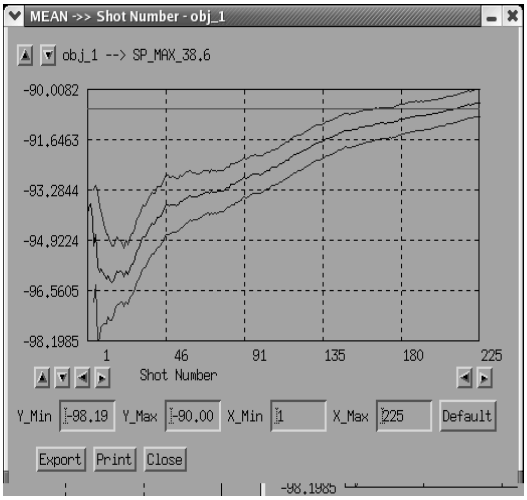

Figure 9: Mean of the objective variable distribution vs. shot number.

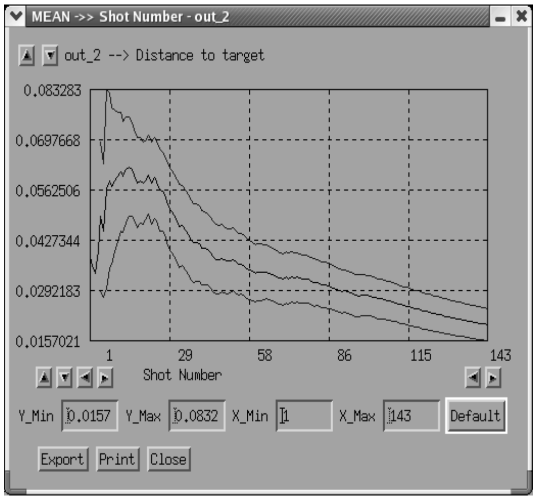

Figure 10: Distance to target vs. shot number.

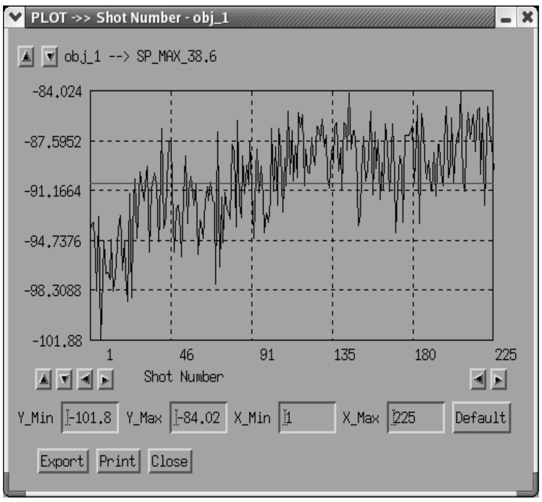

Figure 11: scatter plot of objective variable

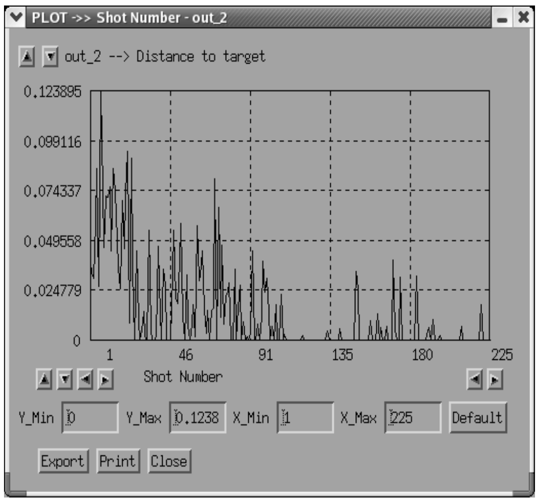

Figure 12: scatter plot of distance to target.

The new nominal value of the design variables after the application of SDI procedure is 1.98 mm for both of them.

A very good agreement of the numerical solution is observed in comparison to the experimental data in the first part of the curve, where they are practically overlapped and where the attention has been focused during the development of numerical simulations necessary to complete the SDI process.

The general conclusion from this study has been that the classical numerical simulations based on nominal values of the input variables are not exhaustive of the phenomenon in the case of crash analyses and can bring to incorrect interpretations of the dynamic behaviour of the examined structure. On the contrary, by using an SDI approach, it is possible to have a better understanding of the influence of each input variable on the structural dynamic behaviour and to assign the most appropriate nominal values in order to have results as near as possible to the target values, also in presence of their natural variability.

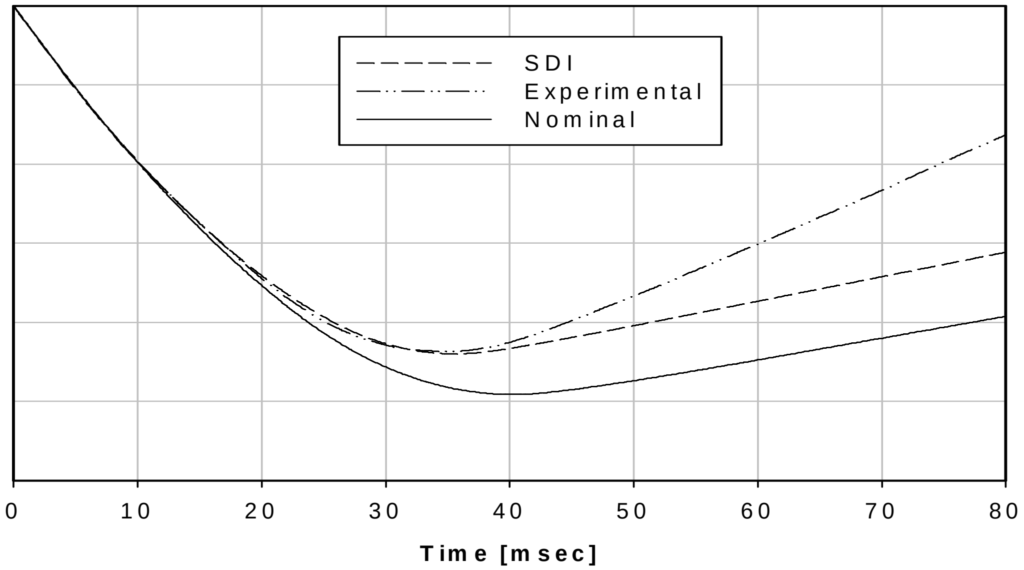

Figure 13: Output variable vs. time

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