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Stress intensity factors for an inclined and/or eccentric crack in a finite orthotropic lamina

Abstract

Stress intensity factors (SIF) are determined for an inclined and / or eccentric crack in a finite orthotropic lamina using the boundary collocation method. The stress functions are defined such that they satisfy governing equations in the domain, the boundary condition on the crack surface and also the singularity at the crack tip. The unknown coefficients in the stress functions are determined such that the boundary condition on the edges of lamina is satisfied. The analysis is also being carried out for isotropic material using finite element analysis software ANSYS for comparison of results. Also, comparison of results with existing solutions is found in good agreement.

Keywords

Stress intensity factors, orthotropic lamina, eccentric crack, boundary collocation.

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NOTATION

$a_{11},a_{12},a_{22},$ a_{16},a_{26},a_{66}	Elastic Constants for orthotropic lamina	σ_x,σ_y	Stresses in x and y directions respectively
P	Applied Stress in y-direction	τ_{xy}	Shear stress in x-y plane.
Q	No. of collocation points on the edges of the lamina	ν_{12}	Poisson's Ratio
G_{12}	Shear Modulus	K_I, K_{II}	Mode I and Mode II Stress intensity Factors respectively

e_x, e_y	Eccentricity of the crack in x and y directions respectively.	Y_I, Y_{II}	Normalized Mode I and Mode II Stress intensity Factors respectively
Re	Real part	N,M	No. of terms associated with the stress functions
α	Angle of fibre orientation	z_k	Complex coordinate, $z_k=x+s_ky$.
ε_x, ε_y, γ_{xy}	Longitudinal, transverse and shear strains	U(x,y)	Airy's stress function
$\Phi(z_1)$, $\psi(z_2)$	Stress Functions	A_k, B_k,	Unknown coefficients in the stress functions.
a	Crack length	ϑ	Angle of the crack.
s_k , k=1,2	Complex parameters of anisotropy	k	Biaxial loading factor

1 INTRODUCTION

The topic of crack problem in finite orthotropic lamina has received considerable attention as the applications of structures using the composite materials are increasing with every sunrise. The SIF of a finite lamina is different from that of infinite lamina according to the dimensions of lamina, crack length, crack angle, material properties and boundary conditions. Accurate determination of stress intensity factor of finite lamina is required for ensuring reliable design of the structures.

Boundary collocation method has been proved to be an effective computational tool and has been successfully applied to variety of crack problems of isotropic materials (Chen and Chen, 1981; Newman, 1976; Ukadgaonkar and Murali, 1991; Wang et.al, 2003) and anisotropic materials (Wang and Chang, 1994; Woo and Samson, 1993). Various methods like modified mapping collocation method (Bowie and Freese, 1972), boundary integral equations (Tan and Gao, 1992) and singular integral equation formulation (Ryan and Mall, 1989) are successfully applied to solve the problems of finite orthotropic lamina / laminates containing edge / central crack.

The finite element method for determination of stress intensity factors is not well suited for discontinuities that do not align with the element edges. In order to get rid of this, mixed mode stress intensity factors using element free Galerkin method (Ghorashi et.al, 2011). Further to this, enriched finite element method is employed (Ozkan et.al., 2010) for study of effect of anisotropy on stress intensity factors for a cracked compact tension specimen, edge specimen and a plate containing central inclined crack. Calculation of the SIF from full field displacement data and asymptotic expansion of the crack tip displacement field is one possibility. This approach requires use of X-ray diffraction and optical microscopy (Wang et.al., 2011). Conformal mapping is promising method for solution of various crack problems in anisotropic plates. It is used (Beam and Jang, 2011) for obtaining SIF of a crack emanating from a infinite wedge in anisotropic material under antiplane shear.

In the present work, stress intensity factors are determined for an inclined crack in a finite orthotropic lamina subjected to uniaxial / biaxial loading to study the effect of fibre angle, crack angle and biaxial loading factor. Also, effect of eccentricity in x direction and/or y direction is studied.

2 BASIC EQUATIONS

The generalized Hooke's Law in plane stress for an orthotropic lamina can be expressed in the following manner (Lekhnitskii, 1976).

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{1}$$

Representing σ_x, σ_y, τ_{xy} in terms of Airy's stress function U(x, y)

$$\sigma_x = \frac{\partial^2 U}{\partial y^2};\, \sigma_y = \frac{\partial^2 U}{\partial x^2};\, \tau_{xy} = \frac{-\,\partial^2 U}{\partial x\, \partial y} \tag{2}$$

The compatibility condition is,

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x\, \partial y} \tag{3}$$

By introducing Hooke's law in terms of F(x,y) into the compatibility equation, we get following biharmonic equation

$$a_{22} \frac{\partial^4 U}{\partial x^4} - 2a_{26} \frac{\partial^4 U}{\partial x^3\, \partial y} + (2a_{12} + a_{66}) \frac{\partial^4 U}{\partial x^2\, \partial y^2} - 2a_{26} \frac{\partial^4 U}{\partial y^3\, \partial x} + a_{11} \frac{\partial^4 U}{\partial y^4} = 0 \tag{4}$$

The Airy's stress functions can be represented as

$$\begin{gathered} U(x, y) = 2\mathrm{Re}[U_1(z_1) + U_2(z_2)] \\ z_1 = x + s_1 y;\, z_2 = x + s_2 y \end{gathered} \tag{5}$$

U_1 and U_2 are analytic functions of complex variables z_1 and z_2 respectively. The complex parameters s_1 and s_2 are roots of characteristic equation

$$a_{11}s^4 - 2a_{16}s^3 + (2a_{12} + a_{66})s^2 - 2a_{26}s + a_{22} = 0 \tag{6}$$

The stress components are represented in terms of stress functions,

$$\sigma_y = 2\mathrm{Re}[\Phi'(z_1) + \psi'(z_2)]$$

$$\begin{aligned}\sigma_x &= 2\mathrm{Re}[s_1^2\Phi'(z_1) + s_2^2\psi'(z_2)]\\ \tau_{xy} &= -2\mathrm{Re}[s_1\Phi'(z_1) + s_2\psi'(z_2)]\\ \text{where, } \Phi(z_1) &= U_1'(z_1), \psi(z_2) = U_2'(z_2)\end{aligned} \tag{7}$$

3 STRESS FUNCTIONS

Figure 1 shows various crack configurations considered in the present study. The coordinate system x_1-o_1-y_1 is used for defining the various orientations. The principal axes of orthotropy are considered to be located at an angle α to the reference axis. The crack is inclined at an angle ϑ. The stress functions are defined with respect to x-o-y coordinate system and hence facilitating further determination of stress intensity factors.

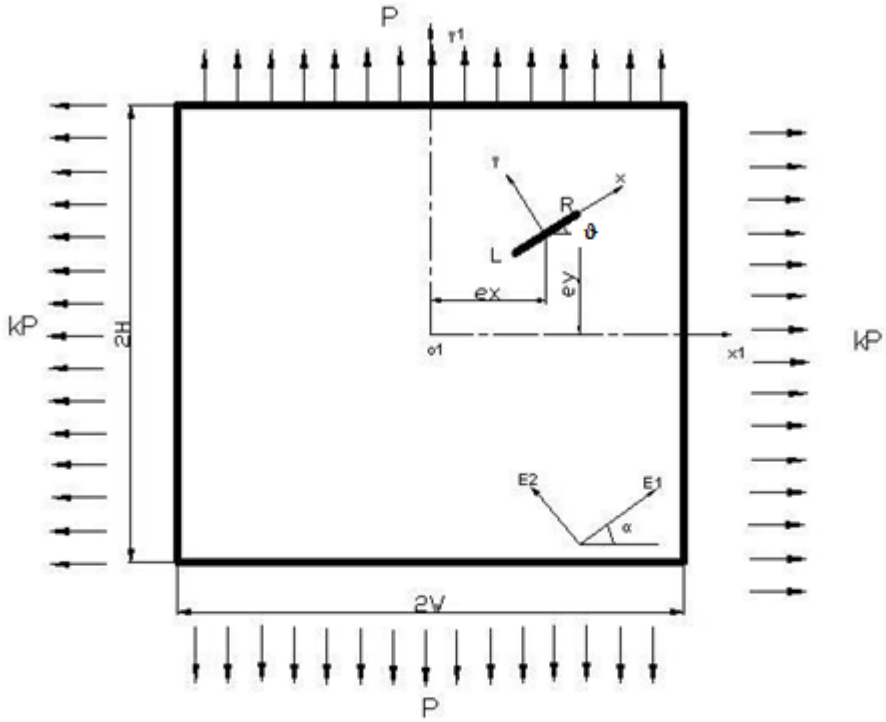

Figure 1: An inclined and / or eccentric crack in a finite orthotropic lamina.

A function F is introduced to define the functions Φ and ψ. The function F consists of

$$F = F_1 + F_2 \tag{8}$$

The functions Φ and ψ consist of two parts viz.

$$\begin{aligned}\Phi &= \Phi_1 + \Phi_2\\ \psi &= \psi_1 + \psi_2\end{aligned} \tag{9}$$

Relationship between Φ_1, ψ_1 and F_1 is

$$\begin{gathered}\Phi_1(z_1) = F_1(z_1)\\ \psi_1(z_2) = -B\bar{F}_1(z_2) + CF_1(z_2)\end{gathered} \tag{10}$$

The relationship between Φ_2, ψ_2 and F_2 is,

$$\begin{gathered}\Phi_2(z_2) = F_2(z_2)\\ \psi_2(z_2) = -B\bar{F}_2(z_2) + CF_2(z_2)\\ B = \frac{(\bar{s}_2 - \bar{s}_1)}{(s_2 - \bar{s}_2)}; C = \frac{(\bar{s}_2 - s_1)}{(s_2 - \bar{s}_2)}\end{gathered} \tag{11}$$

In order to satisfy the stress singularity at the crack tips and ensuring stress free boundary condition on the crack surface, stress functions are defined as,

$$F_1(z) = \sqrt{(z^2 - a^2)} \sum_{n=-N}^{M} A_k\, z^{n-1}$$

$$F_2(z) = \sum_{n=-N}^{M} B_k\, z^{n-1}$$

Leading to,

$$\begin{gathered}\Phi(z_1) = \sqrt{(z_1{}^2 - a^2)} \sum_{n=-N}^{M} A_k\, z_1^{n-1} + \sum_{n=-N}^{M} B_k\, z_1^{n-1}\\ \psi(z_2) = C\left\{\sqrt{z_2{}^2 - a^2} \sum_{n=-N}^{M} A_k\, z_2^{n-1} + \sum_{n=-N}^{M} B_k\, z_2^{n-1}\right\} + B\left\{\sum_{n=-N}^{M} B_k\, \bar{z}_2^{n-1} - \sqrt{(\bar{z}_2^2 - a^2)} \sum_{n=-N}^{M} A_k\, \bar{z}_2^{n-1}\right\}\end{gathered} \tag{12}$$

A_k and B_k are the complex coefficients to be determined such that boundary condition on the edges of lamina is satisfied.

The stress components can be calculated as per equation 7.

At a point on the lamina boundary, boundary condition can be represented as of two types

i) Stress boundary condition

ii) Force boundary condition

Force boundary condition is expressed in terms of

$$f_x + if_y = (1 + is_1)\Phi(z_1) + (1 + is_2)\psi(z_2) + (1 + i\bar{s}_1)\overline{\Phi(z_1)} + (1 + i\bar{s}_2)\overline{\psi(z_2)} + c \tag{13}$$

The determination of constants in the stress functions and hence the stress functions will facilitate the determination of stress intensity factors as,

$$K_{IR} = 2\sqrt{2\pi}\frac{(s_2 - s_1)}{s_2}\lim_{z_1\to a}\sqrt{(z_1 - a)e^{-i\vartheta}}\Phi_1'(z_1)$$
$$K_{IIR} = 2\sqrt{2\pi}\,(s_2 - s_1)\lim_{z_1\to a}\sqrt{(z_1 - a)e^{-i\vartheta}}\Phi_1'(z_1)$$
$$K_{IL} = 2\sqrt{2\pi}\frac{(s_2 - s_1)}{s_2}\lim_{z_1\to -a}\sqrt{(z_1 + a)e^{-i\vartheta}}\Phi_1'(z_1)$$
$$K_{IIL} = 2\sqrt{2\pi}\,(s_2 - s_1)\lim_{z_1\to -a}\sqrt{(z_1 + a)e^{-i\vartheta}}\Phi_1'(z_1) \tag{14}$$

4 NUMERICAL RESULTS AND DISCUSSIONS

4.1 Solution Procedure

1. The coordinates of No. of collocation points Q are marked which along with the angle of the fibre orientation determine the z_1 and z_2.
2. At all collocation points, total error with respect to stresses (equation 7) and traction (equation. 13) is calculated.
3. Let No. of terms in the stress functions =M+N+1, number of unknowns = 2(M+N+1)
4. The coefficients of the stress functions are determined using least square method which will result in 2(M+N+1) number of equations. System of equations is solved using Math CAD.
5. The stress intensity factors are then calculated as per equation 14.

The normalized SIF are expressed where,

$$Y_I = \frac{K_I}{P\sqrt{\pi a}\cos^2\vartheta}$$
$$Y_{II} = \frac{K_{II}}{P\sqrt{\pi a}\sin\vartheta\cos\vartheta}$$

4.2 Convergence

The convergence of the present method is examined for the number of positive and negative terms in the stress functions i.e. M, N and the number of collocation points Q. The cases being considered are a crack eccentric in x-direction with e_x/W=0.2, a/W=0.2 and a/W=0.7. The results obtained are shown in Figure 2 and Figure 3. It is seen that for smaller crack lengths that is a/W=0.2 number of terms in stress functions and the number of collocation points required to converge the solution is less than those for larger crack length a/W=0.7. Based on this study M=20, N=22 and Q=196 are used to obtain the solution.

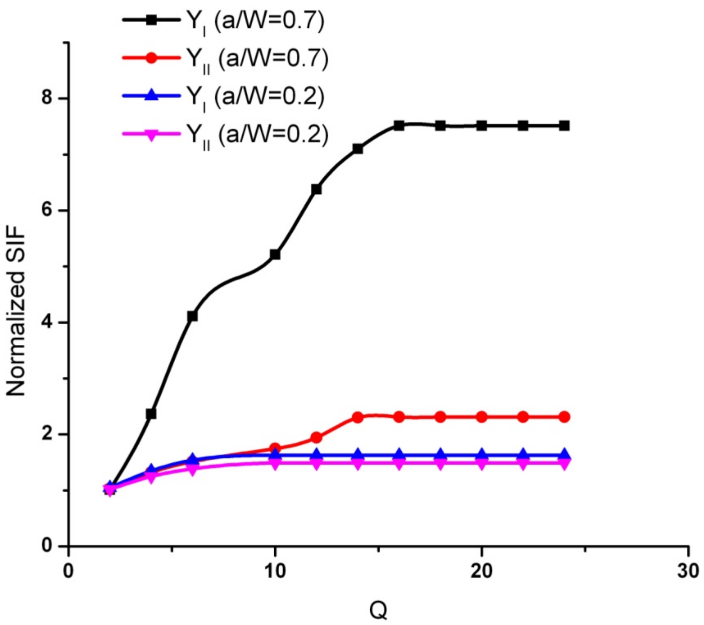

Figure 2: Effect of number of positive and negative terms (M, N) in stress functions on SIF. (Q=196).

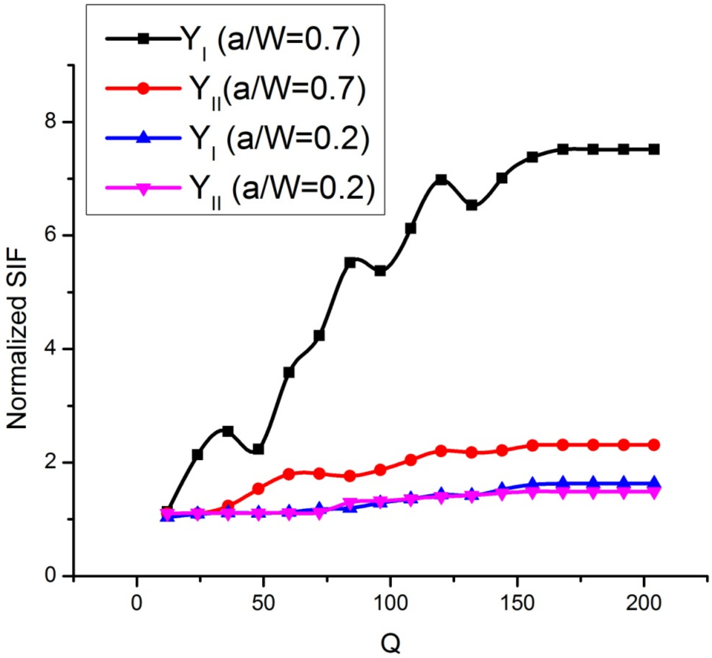

Figure 3: Effect of number of collocation points (M=N=22).

4.3 Comparison of Results

SIF for finite isotropic plate / orthotropic lamina under the uniform stress loading condition are compared with the results already available in the literature and with ANSYS as indicated in Table 1, Table 2, and Table 3.

In ANSYS PLANE 82 elements are used. A concentration keypoint is generated at the crack tip. KCALC command is used for determination of stress intensity factors after the local coordinate system is defined and used at the crack tip and the crack path defined in the post processer using path operations. The model of crack eccentric in x-direction as modeled using ANSYS is shown in Figure 4. It is clear from Figure 5 that quarter point elements exist near the crack tip.

Sr. No.	Configuration	Reference and SIF	ANSYS	Present Method
1	Orthotropic Lamina, E_1=24.13 GPa, E_2=82.74 GPa, G_{12}=20.68 GPa, υ_{12}=0.7. Inclined crack H/W=2, a/W=0.2832 $\vartheta=45^0$	(Wang et. al., 1980) Y_I=1.131 Y_{II} = 1.153	-	Y_I=1.183 Y_{II} = 1.112
3	Isotropic Plate H/W=3, a/W=0.707, Inclined crack $\vartheta=45^0$	(Karami and Fenner, 1986) Y_I=0.732 Y_{II} = 0.591	Y_I=0.798 Y_{II} = 0.637	Y_I=0.764 Y_{II} = 0.613
4	Crack eccentric in x direction, H/W=2.5, a/W=0.2, e_x/W=0.2	(Ukadgaonker and Murali, 1991) Y_{RI}=2.2	Y_{RI} =2.302	Y_{RI}=2.237

Table 1: Comparison of results obtained by present method with those obtained using ANSYS.

a/w→	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Y_I (Bowie and Freese, 1972)	1.01	1.05	1.10	1.18	1.28	1.41	1.59	1.87
Y_I (Present Method)	1.03	1.09	1.17	1.21	1.32	1.49	1.68	1.91

Table 2: Comparison of results for central horizontal crack in orthotropic lamina.

a/ W↓	e_y/W=0		e_y/W=0.2				e_y/W=0.4			
	P	A	P		A		P		A	
	Y_I	Y_I	Y_I	Y_{II}	Y_I	Y_{II}	Y_I	Y_{II}	Y_I	Y_{II}
0.1	1.018	1.016	1.021	0.0006	1.039	0.0002	1.043	0.0018	1.048	0.001
0.2	1.061	1.051	1.072	0.0035	1.067	0.0021	1.110	0.011	1.089	0.015
0.3	1.141	1. 162	1.172	0.012	1.181	0.01	1.203	0.0342	1.197	0.043
0.4	1.251	1.271	1.261	0.0243	1.271	0.0251	1.3712	0.0712	1.354	0.086
0.5	1.401	1.371	1.417	0.0497	1.462	0.051	1.562	0.121	1.617	0.103
0.6	1.512	1.424	1.552	0.0771	1.613	0.078	1.732	0.1867	1.697	0.164
0.7	1.71	1.63	1.812	0.1178	1.834	0.1221	2.105	0.2712	1.981	0.294
0.8	1.82	1.74	2.232	0.1532	2.188	0.1612	2.393	0.3893	2.478	0.433

Table 3: Comparison of results for a central crack and crack eccentric in y direction in finite isotropic plate.

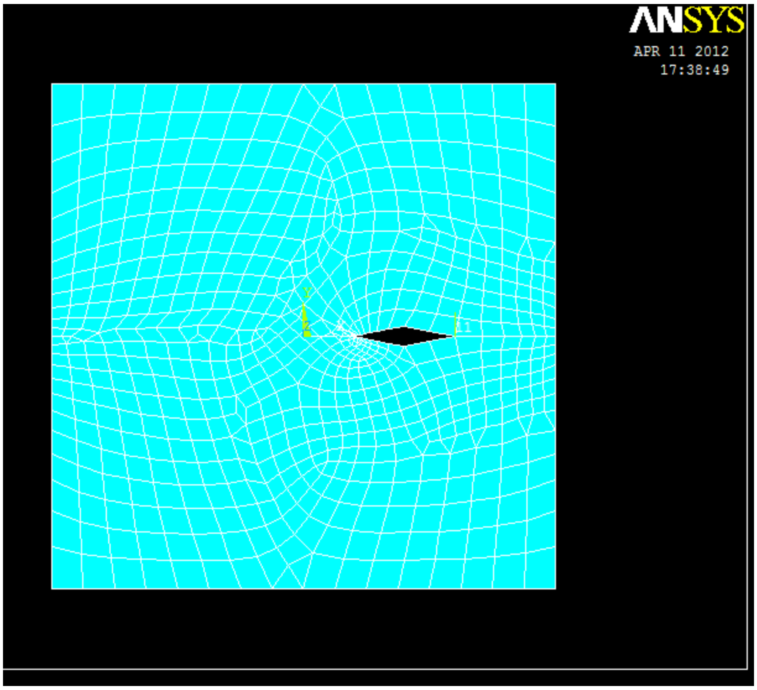

Figure 4: Crack eccentric in X-direction modeled using the ANSYS.

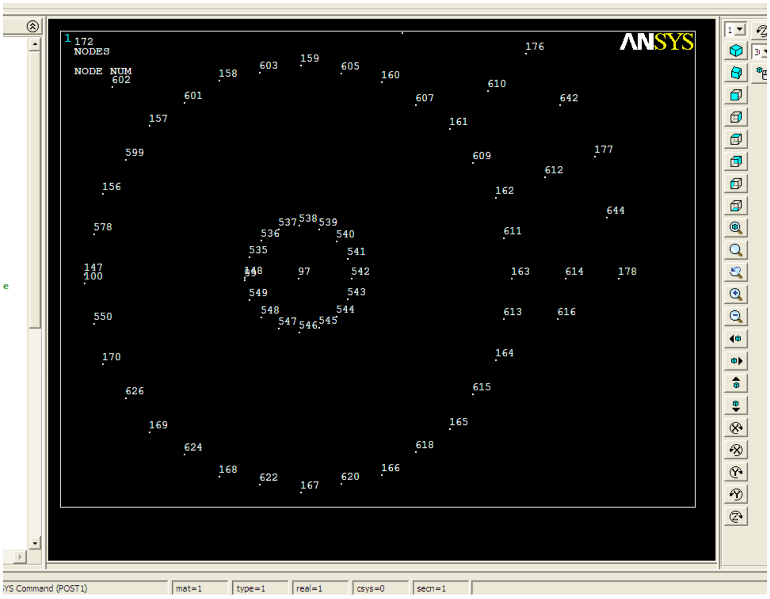

Figure 5: Quarter point elements near crack tip (Node No. 97).

4.4 Case-I –An Inclined Crack under uniaxial loading (k=0).

4.4.1 Effect of fibre angle

Figure 6 shows an inclined crack in a finite orthotropic lamina. Here, k=0 i.e. uniaxial loading is applied, H/W=1, a/W=0.2. The material properties are, E_1=53.74 GPa, E_2=17.91 GPa, G_{12}=8.96 GPa, υ_{12}=0.25

The crack angle is fixed at $\vartheta=45^0$. The fibre angle α is varied from 0^0 to 180^0. The normalized Mode I and Mode II stress intensity factors are plotted in Figure 7. The variation of Y_I and Y_{II} follows same trend with the fibre angle the maxima for both occurs at fibre angle 105^0.

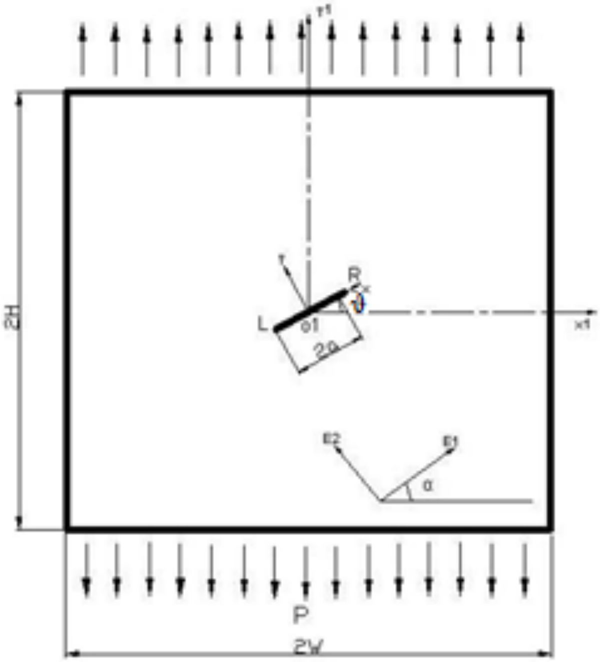

Figure 6: An inclined central crack in a finite orthotropic lamina subjected to uniaxial loading.

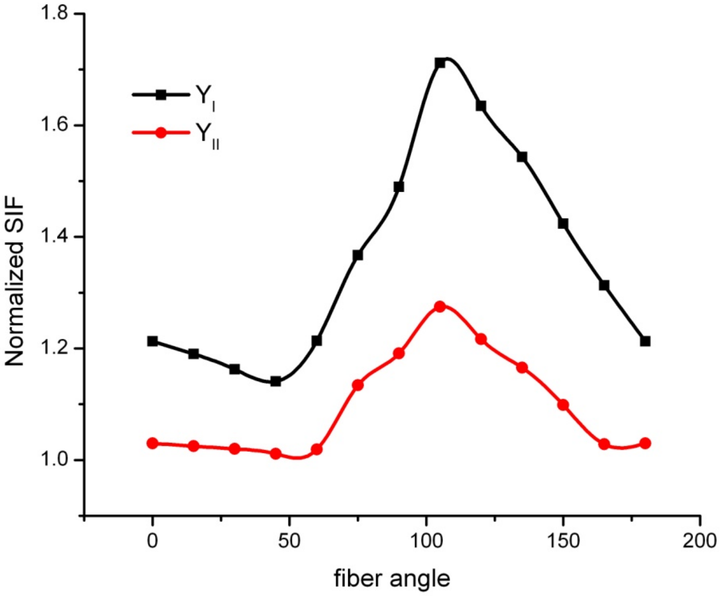

Figure 7: Effect of fibre angle on Mode I and Mode II SIF.

4.4.2 Effect of crack length

The effect of crack length on Y_I and Y_{II} for various combinations of fibre angle and crack angle is plotted in Figures 8, 9. Mode I SIF is highest when both fibre angle and crack angle are zero. If either or both are changed to 45^0, Y_I decreases and percentage decrement decreases with increase in crack length. Mode II SIF values for combinations of fibre angle (α) and crack angle (ϑ) $\alpha=0^0$ $\vartheta=45^0$, $\alpha=45^0$ $\vartheta=0^0$, $\alpha=45^0$ $\vartheta=45^0$ are at par with.

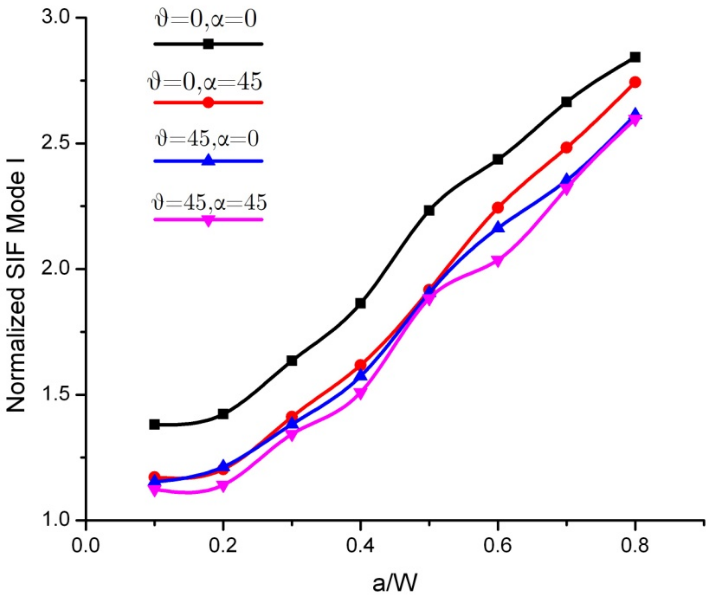

Figure 8: Effect of crack length on Mode I SIF.

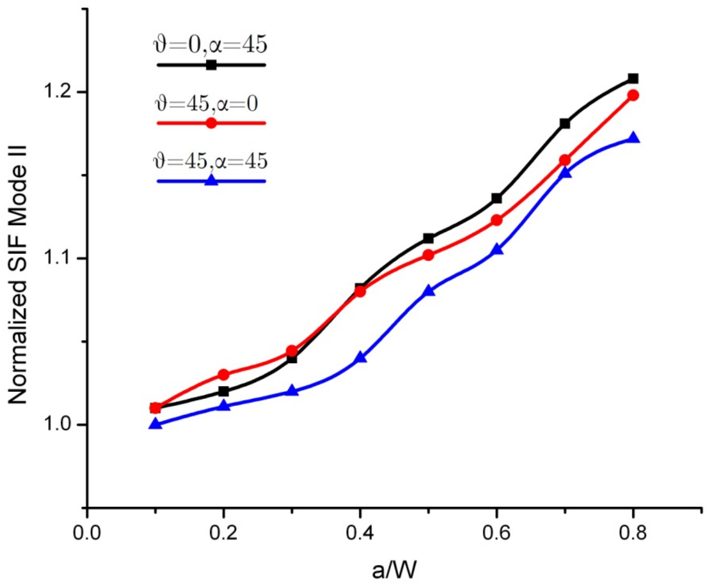

Figure 9: Effect of crack length on Mode II SIF.

4.5 Case-II An inclined crack under the biaxial loading

In order to study the effect of biaxial loading (configuration outlined in Figure 10) on the SIF for an inclined crack, the lamina dimensions are, H/W=1, a/w=0.2 α=0. Material properties are same as those in the previous case. Figure 11, 12 show the variation of Mode I and Mode II SIF respectively with various biaxial loading factors and crack angles. Biaxial loading factor k=1, Mode I SIF values are symmetric about a vertical line passing through crack angle 90^0. For k=1, Mode I SIF is independent of the crack angle. .It is seen that when biaxial loading factor k=1, Y_{II} is independent of crack angle and its value is zero. Mode II SIF is zero for crack angles 0^0,90^0,180^0 irrespective of value of k. Except for k=1 Mode I and Mode II SIF show the sinusoidal variation with maxima/minima occurring at crack angle 90^0 for Mode I SIF and that occurring at 45^0/135^0 for Mode II SIF.

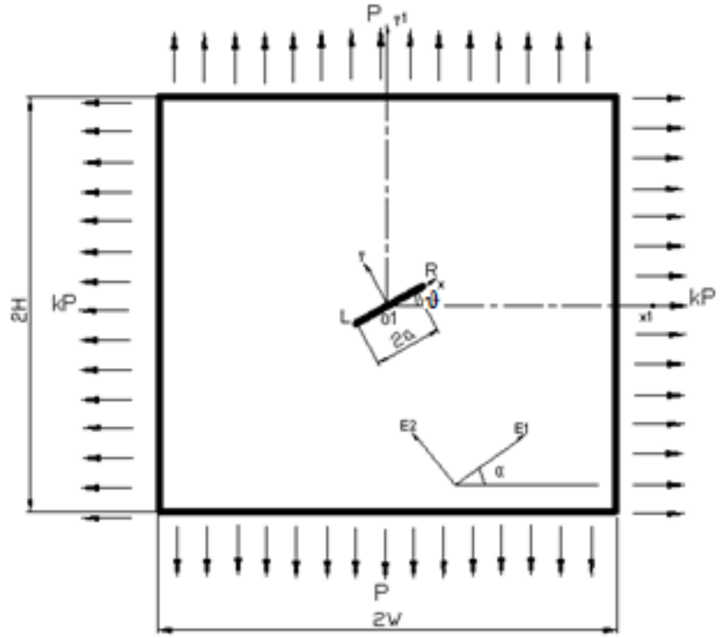

Figure 10: An inclined central crack subjected to biaxial loading.

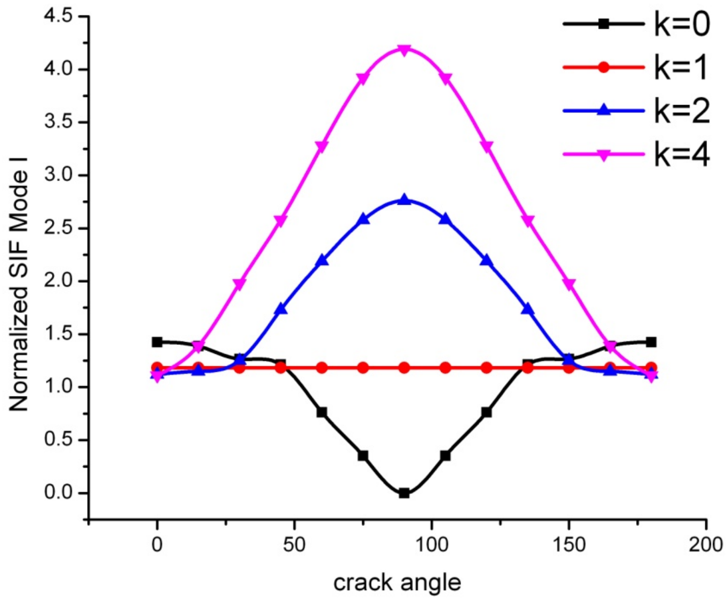

Figure 11: Effect of biaxial loading on Mode I SIF.

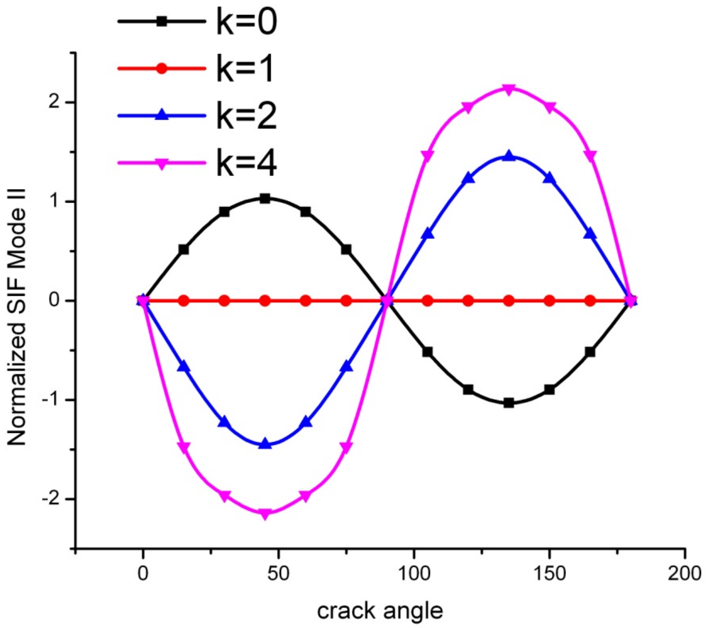

Figure 12: Effect of biaxial loading on Mode II SIF.

4.6 Case III Crack eccentric in y-direction

As shown in Figure 13, a crack is symmetric about the y-axis and situated such that there is a distance from its centre to the plate centre. Here too, H/W=1,$\alpha=0^0$ SIF are calculated for e_y=0, 0.2, 0.4, 0.6, 0.8. For different eccentricities and crack lengths, the stress intensity factors are plotted in Figure 14-15. It is noticed that the Mode I SIF values are same at the both crack tips, and Mode II SIF have different signs but same value. This may be so because, when the plate is in tension, the crack surfaces open. As, the crack is symmetric about the y axis, the two crack tips get opened by the same amount. Because of this phenomenon, Mode I SIF values are same at both the crack tips. The elongation of the lamina in y- direction is accompanied by contraction in x -direction. As the crack is eccentric in y direction, the values of contraction on the two sides besides the crack are un equal. This deformation behavior is the reason that the mode II SIF values exist. By the same reasoning applicable for the Mode I, because the crack is symmetric about the y axis, the Mode II SIF values for the two tips are equal. However, the crack tips are in the first and second quadrants, the deformation in the x-direction has different direction of dislocation, so that the Mode II SIFs have opposite signs. When the crack is very long and the crack tips are near the ends of the plates, the SIF are very large.

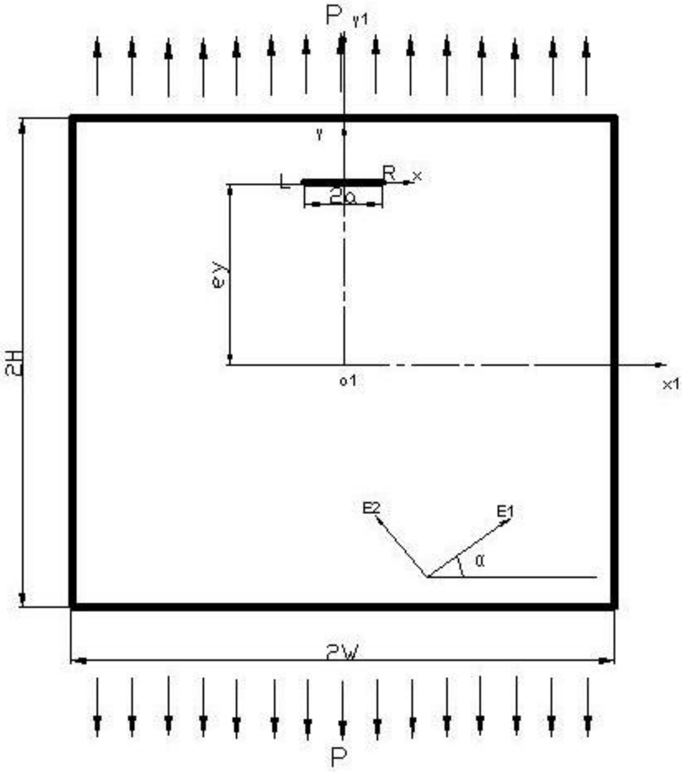

Figure 13: A crack eccentric in y-direction.

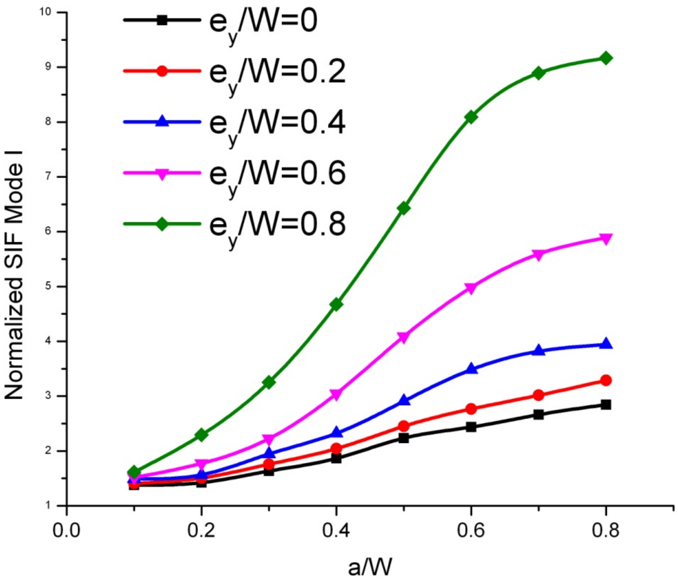

Figure 14: Mode I SIF for crack eccentric in y direction.

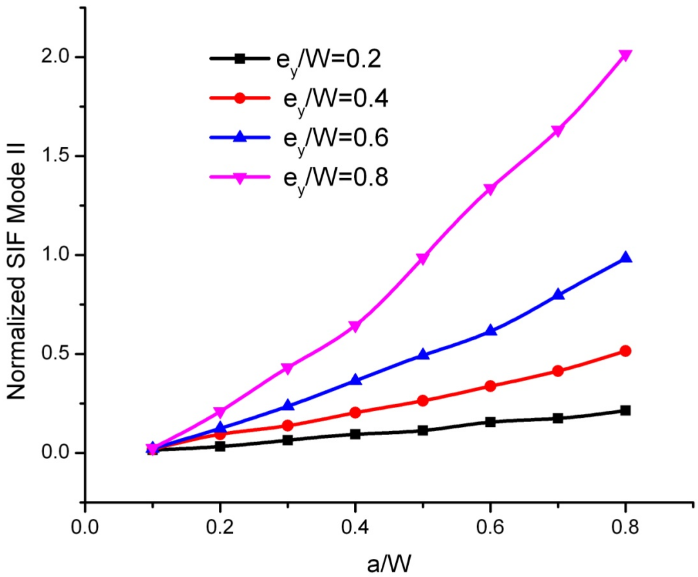

Figure 15: Mode II SIF for crack eccentric in y direction.

4.7 Case IV Crack eccentric in x-direction

Figure 16 shows the of crack eccentric in x-direction with H/W=1, $\alpha=0^0$. Fig. 17 and 18 show the SIF for the right and left crack tips respectively. Here only Mode I SIF exist which are different at the two crack tips. The ratio, SIF at right crack tip to that at left crack tip increases with increase in the crack length. Considerable increment in this ratio is found only after a/w is greater than or equal to 0.3. For a/w greater than 0.5 minimum value of the ratio is 2.5.

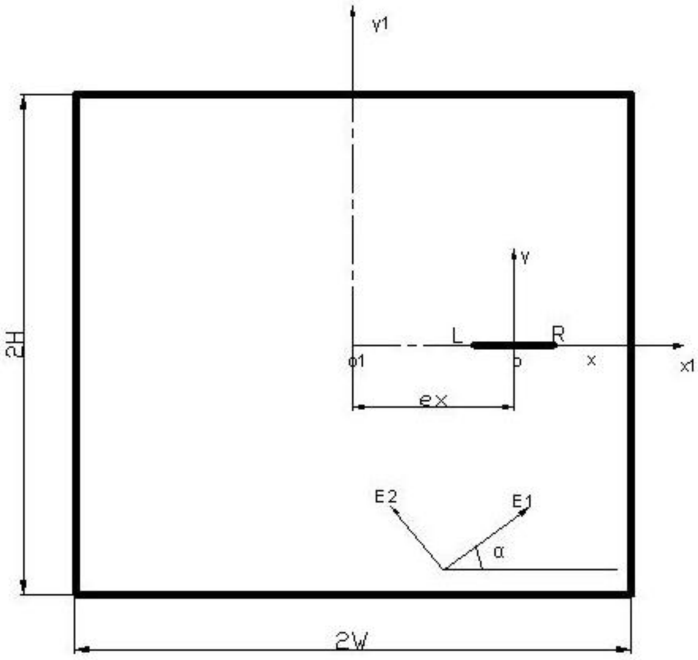

Figure 16: A crack eccentric in x direction.

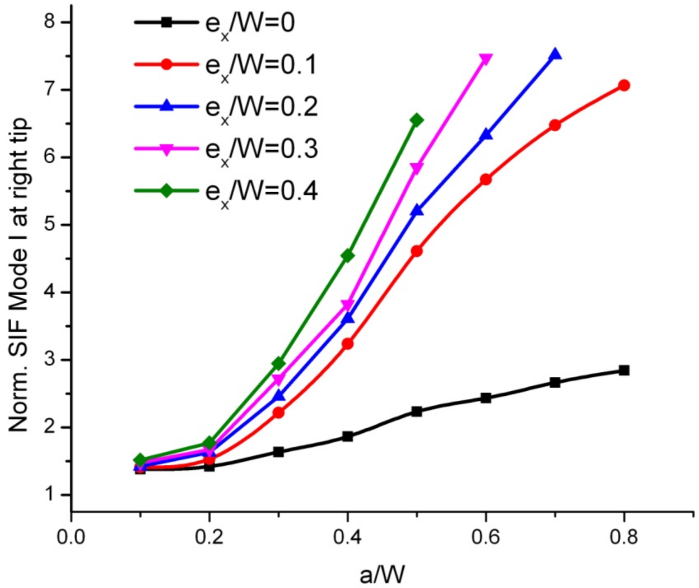

Figure 17: Mode I SIF at right crack tip for crack eccentric in x direction.

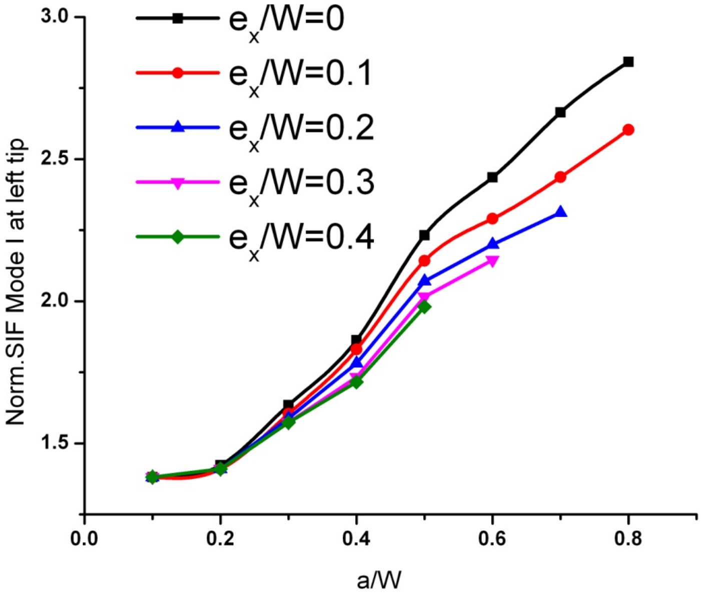

Figure 18: Mode I SIF at left crack tip for crack eccentric in x direct.

4.8 Case V An inclined crack eccentric in both x and y directions

Figure 19 shows the configuration of an inclined crack eccentric in x and y both directions. Here, H/W=1, e_x/W=e_y/W=0.4 and a/W=0.2. Figure. 20 shows the variation of normalized Mode I and Mode II SIF at both crack tips with the crack angle ϑ. The maxima for Mode I occurs at $\vartheta=0^0$, 180^0 and minima occurs at $\vartheta=90^0$, whereas, the maxima for Mode II occurs at $\vartheta=45^0$, minima occurs at $\vartheta=135^0$. The maxima and minima for a given mode occur at the same crack angle for both the tips. Because the crack tips are in the same quadrant, the shear deformation of at the two tips has same sign, so is for Mode II SIF values. The nature of variation coincides with that for k=0 under the study of effect of biaxial loading. Also, the results for H/W=1, $e_x/W=0.1$, $e_y/W=0.4$ and a/W=0.2 are plotted in Fig. 21 indicating that except crack angle = 90^0 , difference in Mode II SIF is higher as compared to results with $e_x/W=e_y/W=0.4$.

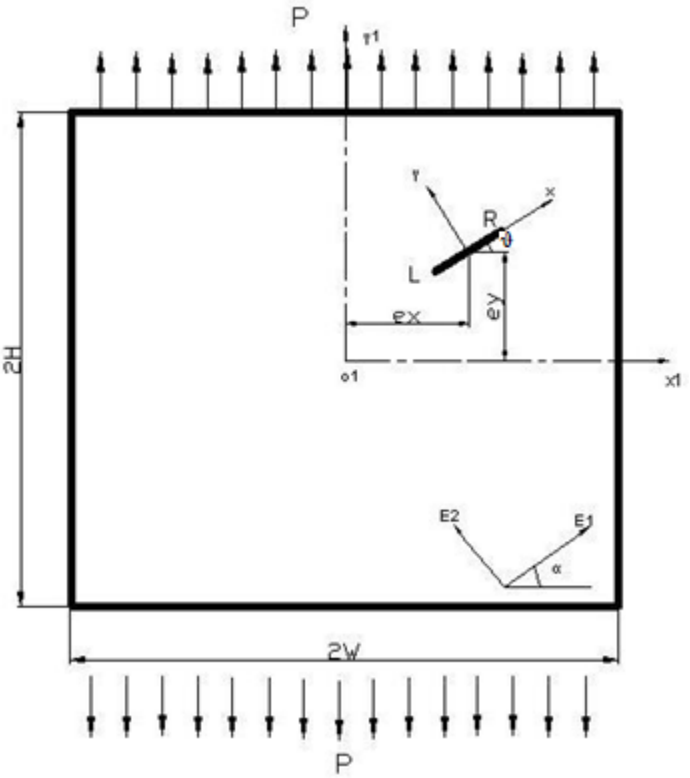

Figure 19: A crack eccentric in both x and y directions.

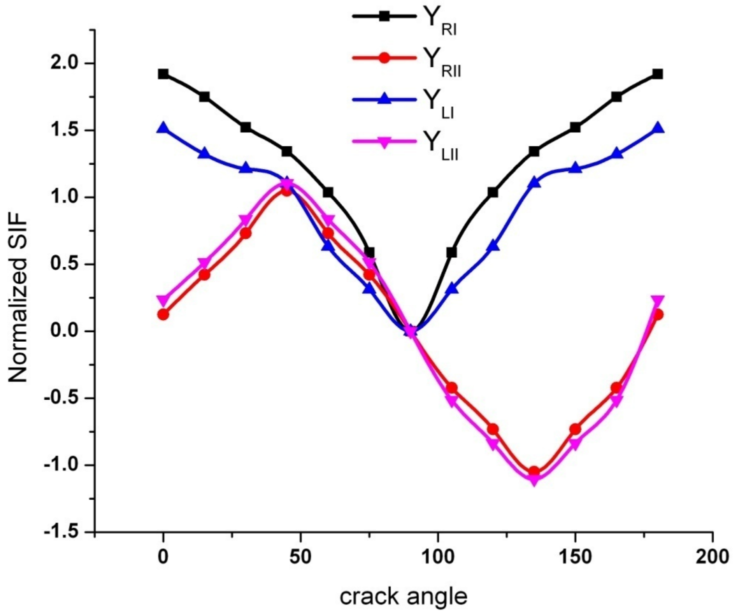

Figure 20: Mode I and Mode II SIF for crack eccentric in both x and y directions with $e_{x/}W=e_{y/}W=0.4$.

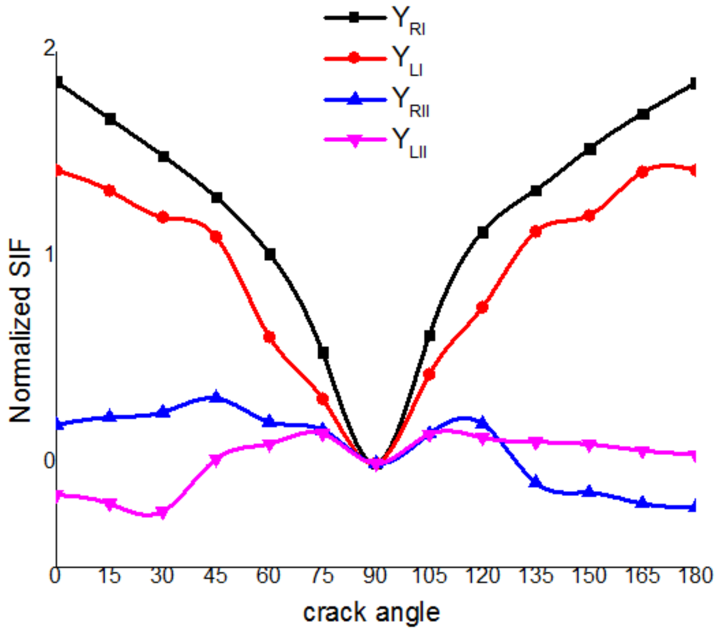

Figure 21: Mode I and Mode II SIF for crack eccentric in both x and y directions with $e_{x/}W=0.1$, $e_{y/}W=0.4$.

5. CONCLUSIONS

Stress functions are defined in this formulation for calculation of Mode I and Mode II SIF of an central crack, inclined crack and crack eccentric in x and / or y direction for finite orthotropic lamina such that governing equations in the domain, stress free condition on the crack surface are satisfied and also, the inverse square root singularity exists at the crack tips. The force type and stress boundary condition on the periphery of the lamina are satisfied by boundary collocation using least square method. The comparison of results with those in literature and with the finite element software ANSYS found in good agreement.

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