ORIGINAL ARTICLE

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Theoretical Investigation on Stiffness and Vibration Characteristics of Laminated Beams with Uniformly Vertical Displacement

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https://doi.org/10.1590/1679-78257035

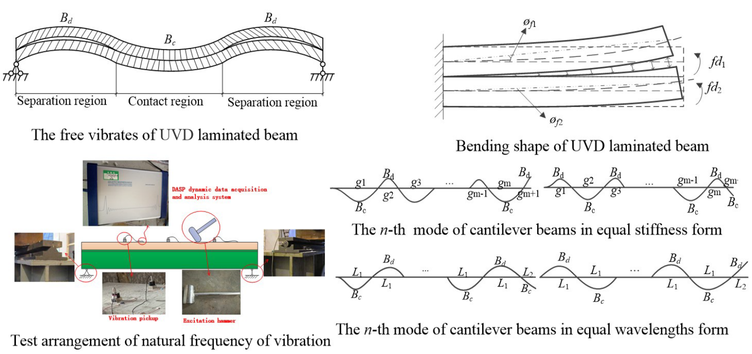
Abstract

The laminated beam with uniformly vertical displacement can be divided into contact region and separation region when bending deformation occurs. Friction between the interfaces will cause the stiffness inconsistency between the separation region and contact region. In this paper, calculation formula of section stiffness considering interface friction effect is derived. Assuming that the beam vibrates freely in equal wavelength and equal stiffness forms respectively, the calculating formulas of natural vibration frequencies of simply supported and cantilever beams are derived. Finally, based on a steel-concrete laminated test beam with uniform vertical displacement, the natural vibration frequencies of the beam are calculated. The conclusions are as follows: The derived formulas can calculate the natural vibration frequencies of laminated beams under different interface states effectively; The influence of friction effect on the vibration frequency of laminated beams becomes more and more obvious with the increase of the order.

Keywords

Laminated beam with uniformly vertical displacement; Friction effect; Bending stiffness; Transfer matrix; Natural vibration frequency

Graphical abstract



Received: March 11, 2022. In revised form: June 24, 2022. Accepted: July 29, 2022. Available online: August 04, 2022
https://doi.org/10.1590/1679-78257035

1 INTRODUCTION

With the excellent mechanical properties, high strength-to-weight ratios, long fatigue life and other superior material properties, laminated beams have been widely used in engineering areas (Mario et al., 2021; Lu et al., 2021; Yasin et al., 2020). In civil engineering, steel beams with good tensile performance and concrete with strong compressive performance are usually combined to achieve the goal of making full use of the excellent properties of materials and reducing costs (Karam et al., 2017; Nie, 2020).

In the last decades, the stress distribution, bearing capacity et al. mechanical properties of laminated beams has been extensive researched (Gao and Wu, 2007; Sun, 2004; Li, 2002; Liu et al., 2021; Sedighi et al., 2013; Huang, 2015; Li and Liu, 2021). Based on the free-laminated beams, the calculation method of bending moment when each layer beam is compacted was proposed by Jie (Jie, 1995). Zhao et al. (Zhao et al., 2008) analyzed the normal stress distribution law of the steel and steel laminated beam, steel and aluminum laminated beam under pure bending conditions. Beyond that, the theoretical calculation formula was derived.

Furthermore, many scholars have analyzed the vibration characteristics of laminated beams. Jiang et al. (Jiang, et al. 2021) analyzed the free vibration characteristics of rotating composite laminated beams under various boundary conditions. Considering the effect of crack, the vibration behavior of a C-F rotating composite laminated beam was investigated by Kim et al. (Kim, et al. 2018). Based on the first-order shear deformation theory, the analysis model of the vibration of the laminated composite beam with arbitrary boundary conditions considering the curing deformation is established by Peng et al. (Peng, et al. 2021), and the natural frequencies are obtained by Rayleigh-Ritz method. The effects of various parameters such as fiber angle and delamination length on the natural frequencies and the mode shapes are studied by Shams, et al. (Shams, et al. 2021). The analyse results by Minh-VanThai et al. (Thai, et al. 2021) show that there is a significant difference between experimental and theoretical fundamental frequencies, and modal damping is negatively correlated with the number of grooves in the beam. Based on the shear-deformable thirteen degrees-of-freedom finite element model, which considers extension-twist, bending-twist and bending-extension couplings, and the Poisson's effect, the free vibration analysis of laminated composite beams including open transverse cracks is presented by Kahya et al. (Kahya, et al. 2019).

In literature (Sarparast and Ebrahimi-Mamaghani, 2019), the forced and free vibrations of a laminated curved beam under moving loads are analyzed, meanwhile the detailed parametric analysis is performed to clarify the influence of various parameters such as stacking sequence and load speed on the vibrational behavior of the system. Free vibration characteristics and buckling behaviors of generally laminated composite beams subjected to the concentrated axial force and various classical end conditions are investigated by Li et al. (Li, et al. 2016). S.K.Sahu and P.Das.(Sahu and Das, 2020) investigated the vibration characteristics of a laminated composite beam (LCB) with multiple transverse cracks, and the natural frequencies of the LCB are computed through the eigenvalue solver.

From what has been discussed above, the static mechanical and vibration characteristics of laminated beams have been thoroughly studied. However, most of above studies were carried out on the assumption that the each layer beam is compacted. Test results show that the vertical lift may appear between different layers of beams when bending deformation occurs in pure laminated beams, which would lead to the significant reduction of beam stiffness and bearing capacity. In view of this phenomenon, a new type of connector was proposed: uplift restricted-slip permission connector (Referred to as “URSP connector”) (Nie, et al. 2015), whose construction is shown in Figure 1. The application of URSP connectors creatively solves the problem of vertical lifting of different layers beams. The URSP connectors are arranged sparsely in the laminated beams, so that the mechanical properties of different layers beams can be effectively developed under the premise of the same vertical displacement. This kind of beam is defined as the laminated beams with uniformly vertical displacement, referred to as UVD laminated beam.

In the UVD laminated beam, when bending deformation occurs, the contact region and separation tendency region (referred to as "separation region") will appear in the beam which has inconsistent stiffness layer beams. In contact region, the tension of connectors is zero, the two adjacent layer beams squeeze each other, and there is friction force between the interfaces. In separation region, the connectors are strained, and there is no squeeze and friction force between the interfaces.

In this paper, focus on a two layer UVD laminated beam, the calculation formula of section stiffness considering the friction between interfaces will be derived. Assuming that the UVD laminated beam vibrates freely at equal wavelength and equal stiffness forms respectively, the formulas of natural vibration frequencies would be derived by using matrix transfer method.

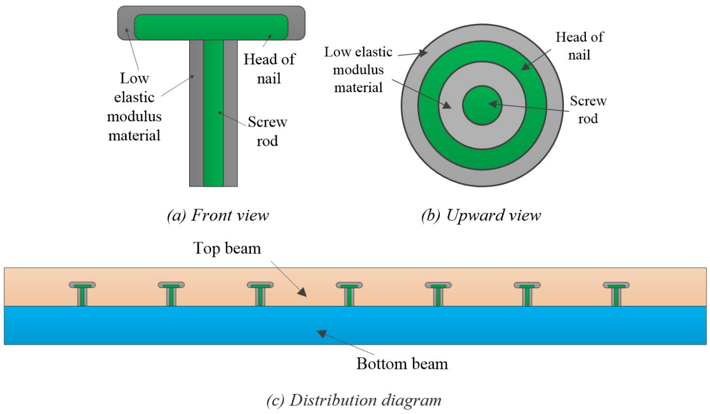


Figure 1. The construction of URSP connector

2 CROSS-SECTION STIFFNESS

Section stiffness refers to the ability of a structure to resist changes in its curved shape. It is the basis for analyzing the deflection and deformation of a structure under load and the ductility of the structure, and is also used in analyzing the natural vibration characteristics. The section stiffness calculation formulas of laminated beams without considering the friction effect have been given in the literature (Liu, 2004). However, in contact region, the friction force in the laminated interface will hinder the slippage from appearing and increase the stiffness of beams. In this section, focus on a two layer UVD laminated beam, the calculation formula of section stiffness of laminated beams considering interfacial friction is deduced.

The micro-segment analysis model of UVD laminated beam in contact region is shown in Figure 2. B_1, E_1, A_1 and I_1 are the stiffness, elastic modulus, cross-sectional area and moment of inertia of the top beam, respectively. The relevant parameters of the bottom beam are B_2, E_2, A_2 and I_2, respectively.

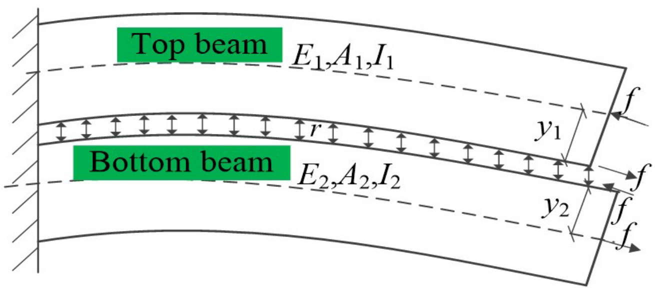


Figure 2 Micro-segment analysis model of UVD laminated beam

Under the action of friction force f, the resistance bending moments M_{f1} and M_{f2} generated at the neutral axis positions of the top and bottom beams, which can be expressed as:

$$M_{fi} = fy_i \tag{1}$$

where:

i=1,2, the y_1 and y_2 are the distances from the neutral axis of the top and bottom beams to the laminated interface respectively.

$f=Q\cdot\mu$: Q is the uniformly loads, μ is the coefficient of friction between interfaces.

The bending curvature ϕ_{fi} generated by the resistance bending moments M_{f1} and M_{f2} are:

$$\varphi_{fi} = \frac{M_{fi}}{E_i I_i} \tag{2}$$

In general, the stiffness of top beam is different from that of bottom beam in laminated beam, which results in $\phi_{f1} \neq \phi_{f2}$. In Figure 3, when $\phi_{f1} > \phi_{f2}$, the top and bottom beams will separate under the resistance bending moment. On the premise of the uniformly vertical displacement of the two layer beams, the curvature of the two layer beams is also the same. The smaller one of ϕ_{f1} and ϕ_{f2} can be taken as the influence of the resistance bending moment on the bending curvature of cross-section, defined as ϕ_m(the ϕ_m can be determined according to the stiffness and deformation of different layers beams).

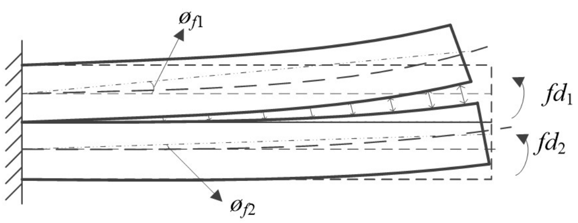


Figure 3 Bending shape of UVD laminated beam

The resistance strains ε_1 and ε_2 caused by the interfacial friction force f are:

$$\varepsilon_i = \frac{f}{E_i A_i} \tag{3}$$

The bending curvature caused by the resistance strain is ϕ_f', expressed as:

$$\varphi_f^{'} = \frac{\varepsilon_1 + \varepsilon_2}{y_1 + y_2} \tag{4}$$

Combining equation (3) and (4), the resistance curvature of cross section generated by friction resistance is ϕ:

$$\varphi = \frac{f}{y_{sc}}\left(\frac{1}{E_1 A_1} + \frac{1}{E_2 A_2}\right) + \varphi_m \tag{5}$$

where the $y_{sc}=y_1+y_2$.

The additional stiffness K_f caused by f can be expressed as:

$$K_f = \frac{f}{y_{sc}}\left(\frac{1}{E_1 A_1} + \frac{1}{E_2 A_2}\right) + \varphi_m \tag{6}$$

The sectional bending stiffness in the negative bending moment region, B can be expressed as:

$$B = B_1 + B_2 + K_f \tag{7}$$

The section stiffness of the UVD laminated beam is determined by the stiffness of the top and bottom beams, and the interface friction coefficient. The greater the friction coefficient, the greater the section stiffness of the beam.

3 FREQUENCY AND MODE

Focus on a two layer UVD laminated beam, when it vibrates periodically, taking 1.5 times wavelength vibration as an example (as shown in Figure 4), the beam alternately appears the separation region and the contact region. In separation region, the section stiffness is the sum of B_1 and B_2, expressed as B_d, in the contact region, the section stiffness is the sum of B_1, B_2 and K_f, expressed as B_c (*ie*. equation (7)).

Before analyzing, the following assumptions are made:

(1) The contact region and separation region appear alternately, showing a periodic distribution;

(2) The vertical deflection of the top and bottom beams is consistent, and the positive pressure on the contact surface is evenly distributed, so the friction resistance is also evenly distributed.

(3) The laminated beam vibrates in two forms of equal wavelength and equal stiffness respectively.

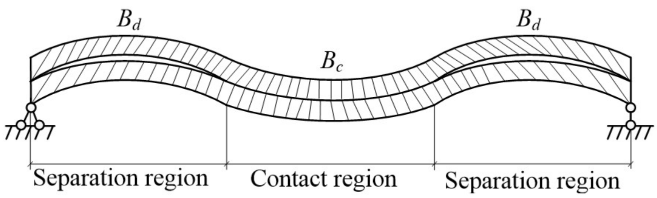


Figure 4 The free vibrates of UVD laminated beam

3.1 Free vibrate at equal wavelengths form

It is assumed that the UVD laminated beams vibration in equal wavelength form, *ie*, the wavelengths in the modes are all equal. The separation region and contact region are distributed in two adjacent wavelengths of the beam. Meanwhile, the separation region and contact region correspond to different stiffness, so the N-th vibration mode of the beam has two stiffness distribution forms, as show in Figure 5.

When the laminated beam vibrates periodically according to the law shown in Figure 5, the beam of length L is divided into n segments with equal length, as show in Figure 6. And then, the natural vibration frequency of the beam can be calculated by the method of transfer matrix. In Figure 6, the 1, 2, ⋯, i, i+1, ⋯, n is the number of beam segment, and the corresponding length of each beam segment is d, $d=L/n$。

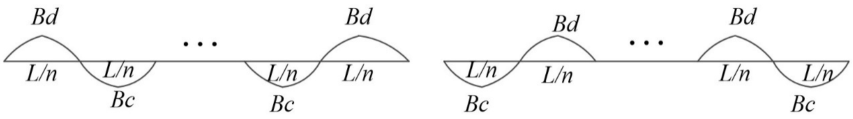


Figure 5 The *n*-th order mode of beam in equal wavelength

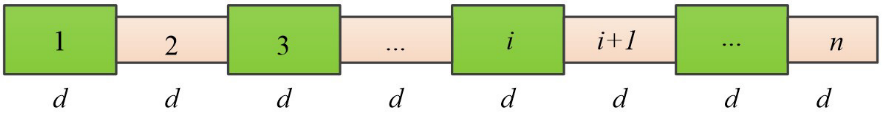


Figure 6 *N* segments beam with variable stiffness

It is assumed that the bending stiffness of the *i*-th segments of the UVD laminated beam is B_i, the mass per unit length of the beam is *m*. According to Euler-Bernoulli beam theory (Krogh, et al., 2006), the mode function of the *i*-th beam segment is:

$$\varphi_i(x) = C_1^i \sin\beta_i x + C_2^i \cos\beta_i x + C_3^i \sinh\beta_i x + C_4^i \cosh\beta_i x \tag{8}$$

where the C_k^i is the undetermined coefficient of vibration mode, *k*=1,2,3,4

$$\beta_i^4 = \frac{m\omega^2}{B_i} \tag{9}$$

For the two adjacent beam segments, the right end of the *i*-th beam segment and the left end of the (*i*+1)-th beam segment satisfy the conditions of displacement and rotation angle continuity. At the same time, the shear force, bending moment balance conditions is satisfied, which can be represented as:

$$\begin{cases} \varphi_i(d) = \varphi_{i+1}(0) \\ \varphi_i'(x)\big|_{x=d} = \varphi_{i+1}'(x)\big|_{x=0} \\ B_i\varphi_i''(x)\big|_{x=d} = B_{i+1}\varphi_{i+1}''(x)\big|_{x=0} \\ B_i\varphi_i'''(x)\big|_{x=d} = B_{i+1}\varphi_{i+1}'''(x)\big|_{x=0} \end{cases} \tag{10}$$

Convert Eq. (10) to matrix form, *i.e*:

$$\mathbf{Q}_i\mathbf{C}^i = \mathbf{Q}_{i+1}\mathbf{C}^{i+1} \tag{11}$$

Equation (11) can be rewritten as:

$$\mathbf{C}^{i+1} = \mathbf{Q}_{i+1}^{-1}\mathbf{Q}_i\mathbf{C}^i = \mathbf{T}_i\mathbf{C}^i \tag{12}$$

where the T_i is the transfer matrix of undetermined coefficients between the *i*-th beam segment and the (*i*+1)-th beam segment. According to equation (11), the transfer relation of undetermined coefficients of vibration modes between the last beam segment and the first beam segment can be deduced by recursive method, *i.e*:

$$\mathbf{C}^n = \mathbf{T}\mathbf{C}^1 \quad (13)$$

where $\mathbf{T} = \mathbf{T}_{n-1}\mathbf{T}_{n-2}\cdots\mathbf{T}_2\mathbf{T}_1$

From the boundary conditions at both ends of the laminated beam, there is:

$$\mathbf{D}\mathbf{C}^1 = \begin{bmatrix} \mathbf{D}_L \\ \mathbf{D}_R\ \mathbf{T} \end{bmatrix}\mathbf{C}^1 = 0 \quad (14)$$

where **D** is the characteristic vector corresponding to the vibration mode, the matrices $\mathbf{D}_L$ and $\mathbf{D}_R$ are respectively:

$$\mathbf{D}_L = \begin{bmatrix} \sin(0) & \cos(0) & \sinh(0) & \cosh(0) \\ -\sin(0) & -\cos(0) & \sinh(0) & \cosh(0) \end{bmatrix} \mathbf{D}_R = \begin{bmatrix} \sin(k_n l) & \cos(k_n l) & \sinh(k_n l) & \cosh(k_n l) \\ -\sin(k_n l) & -\cos(k_n l) & \sinh(k_n l) & \cosh(k_n l) \end{bmatrix}$$

3.1.1 Simply supported laminated beam

For the simply supported beam, the first beam segment and the *n*-th beam segment satisfied the follow boundary conditions:

$$\begin{cases} \varphi_1\left(x=0\right) = B_{1,i}\dfrac{d^2\varphi_1}{dx^2} = 0 \\ \varphi_1\left(x=l\right) = B_{n,i}\dfrac{d^2\varphi_n}{dx^2} = 0 \end{cases} \quad (15)$$

The boundary conditions of equation (15) can be written in the same matrix form as equation (14). By solving the matrix characteristics, the natural vibration frequency of simply supported beams vibrating freely in equal wavelength form (namely, the eigenvalue of **D**) can be obtained. Then the eigenvectors are solved from the eigenvalues, the undetermined coefficient of the first beam segment ($\mathbf{C}^1$) can be obtained.

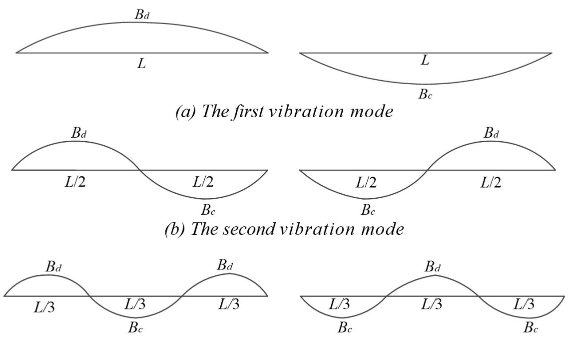


Figure 7 The first three vibration modes of simply supported beams vibrate in equal wavelengths

The vibration mode of a simply supported beam vibrating in equal wavelength form can be obtained by substituting the undetermined coefficient C1 into equations (8) and (12). The first three vibration modes of a simply supported beam when it vibrates freely in equal wavelengths form are shown in Figure 7.

As shown in Figure 7, the beam presents two distribution patterns around the equilibrium position when vibrates freely in equal wavelength form. Because the stiffness of contact region is different from that of separation region, the frequency of beams are different. When solving the frequency of even order modes, the stiffness presents an antisymmetric distribution, the beam has only one natural frequency; while, when solving the frequencies of odd order modes, the stiffness has two distribution forms, so the beam corresponds to two frequencies at the same time.

3.1.2 Cantilever laminated beam

The cantilever laminated beam is shown in Figure 8, the corresponding first order and *N*-th order modes of the cantilever beam vibrating freely in equal wavelength form are shown in Figure 9.

Figure 8 Cantilever laminated beam

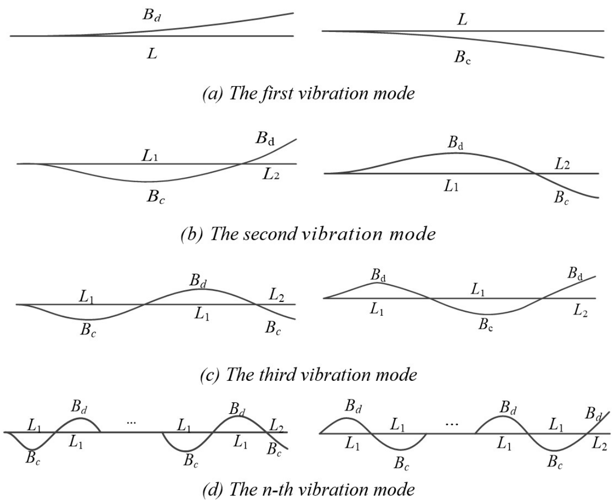


Figure 9 The vibration modes of cantilever beams in equal wavelengths form

When solving the *n*-th natural vibration frequency of the cantilever beam, the beam with length *L* is divided into *m*+1 segments, the length of the first *m* segment is *g*,the length of (*m*+1)-th is *g'*, *g*=(*L*-*g'*)/*m*. The stiffness distribution of the cantilever beam has two forms as shown in Figure 10, in which, the stiffness of the first beam segment is B_m(B_m=B_c/B_d),the mass of per unit length is m_0.

The boundary condition: the displacement and angle of the cantilever end and the bending moment of the free end are zero, angle of the adjacent beam segment is consistent:

$$\left.\begin{array}{l} \varphi_1(x=0)=0, \varphi_1^{'}(x=0)=0 \\ \varphi_m(x=g)=0, \varphi_{m+1}(x=0)=0 \\ M_{m+1}(x=g)=EI\varphi_{m+1}^{''}(x=g)=0 \end{array}\right\} \tag{16}$$

The equation (16) can be written as matrix form, in order to solve the natural frequency and mode of cantilever beam. When the cantilever beam vibrates freely, because the stiffness of contact region is different from that of separation region, the beam corresponds to two natural frequencies simultaneously.

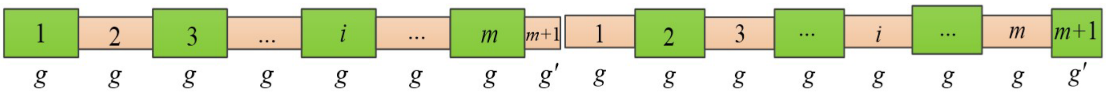


Figure 10 The variable stiffness cantilever beam of *m*+1 segment

3.2 Free vibrate in equal stiffness form

Assuming that the beam vibration freely in equal stiffness form, each beam segment in the vibration mode corresponds to same linear stiffness *K*. The linear stiffness of the beam segment and the bending stiffness of cross-section satisfy the relation equation:

$$K_i \propto \frac{B_i}{d_i^4} \tag{17}$$

where, K_i is the linear stiffness of beam segment; d_i is the length of corresponding beam segment;the value of B_i takes B_d or B_c according to the different interface state of the beam segments.

Different bending states of the beam corresponds to different stiffness distribution, so that the length (wavelength) of beam segment is different. When the laminated beam vibrates freely in equal stiffness form, the vibration mode of the *n*-th order is shown in Figure 11.

Figure 11 *N*-th order mode of beam vibration at equal stiffness form

According to the assumptions and the mode diagram in Figure 11, the following equation is set up:

$$\begin{cases} L_1 + L_2 + \cdots + L_i + L_{i+1} + \cdots + L_n = L \\ \dfrac{B_{1i}}{L_1^4} = \dfrac{B_{2i}}{L_2^4} = \cdots = \dfrac{B_{ji}}{L_j^4} = \cdots = \dfrac{B_{ni}}{L_n^4} \\ L_1 = L_3 = \cdots = L_{2m-1}, L_2 = L_4 = \cdots = L_{2m} \end{cases} \tag{18}$$

Referring to the solution process of natural vibration frequency when the beam vibrates freely in equal wavelength form, equation (18) is combined to solve the natural vibration frequency of laminated beam vibrates freely in equal stiffness form.

3.2.1 Simply supported laminated beam

The first three vibration modes of the simply supported beam when it vibrates freely in equal stiffness form is shown in Figure 12. Combining the boundary conditions of simply supported laminated beam, which are given in equation (15), the natural vibration frequencies can be calculated. Each vibration mode of the beam corresponds to two natural frequencies.

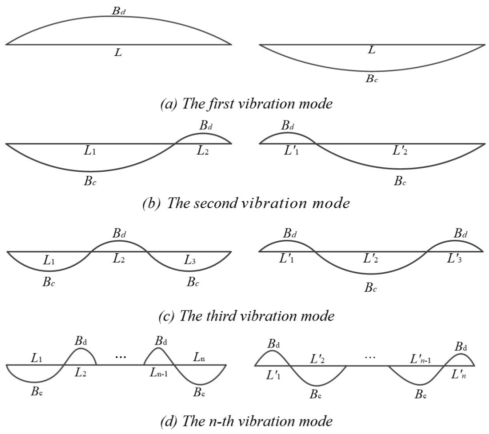


Figure 12 The vibration modes of simply support beams in equal stiffness form

3.2.2 Cantilever laminated beam

When the cantilever beam vibrates freely in equal stiffness form, the first three and *n*-th vibration modes are shown in Figure 13. Each vibration mode corresponds to two kind of stiffness distributions, therefore two natural frequencies. The transfer matrix method is used to solve the vibration characteristics of the cantilever beam, the solving process is same to **section 3.1**. When n=1, the natural vibration frequency is solved according to the solution method for equal wavelength vibration.

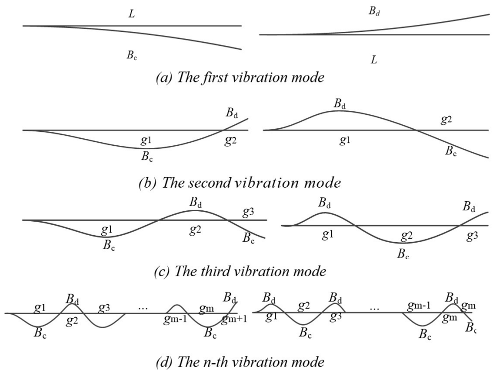


Figure 13 The vibration modes of cantilever beams in equal stiffness form

4 NUMERICAL EXAMPLE

Take a steel-concrete composite-laminated simply supported beam (Referred to as “CLB”) as example, the calculated parameters are shown in Figure 14 (a) and (b). The top concrete slab and the bottom composite beam (composite beam of bottom concrete and steel profile) are connected by URSP connectors.

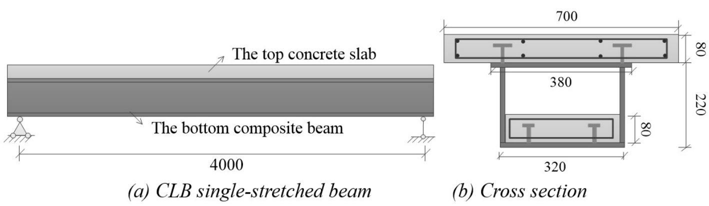


Figure 14 Calculation sketch of CLB(mm)

The steel beam was welded by 8mm thick Q345 steel plate, the longitudinal stress reinforcement is ϕ8 HRB400 grade reinforcement, and the concrete adapt C40 grade concrete. The material parameters is shown in Table 1:

Table 1 Material parameters (MPa)

	Steel			Concrete		
	f_t	f_u	E_s	f_c	f_{t0}	E_c
Steel plate	385	460	2.1×10^5	26.8	2.39	3.25×10^4
Steel rebar	435	580	2.1×10^5			

Note: f_t, f_u, Es are the yield strength, ultimate strength and elastic modulus of steel respectively; f_c, f_{t0} and E_c are the compressive strength, tensile strength and elastic modulus of concrete respectively.

The section stiffness of the top beam is calculated by the conversion cross section method: $B_1=9.7\times10^{11}$N·mm^2.

The field layout of vibration test of steel-concrete composite-laminated simply supported beam is shown in the Figure 15. Experimental instruments mainly include model 941B vertical vibration collector of COINV and DASP 10.0 dynamic data acquisition and analysis system of COINV. Test dynamic trigger equipment adopt small DFC excitation hammer.The vibration test measured the first-order natural frequencies at three different interface states. The different interface states are as follows:

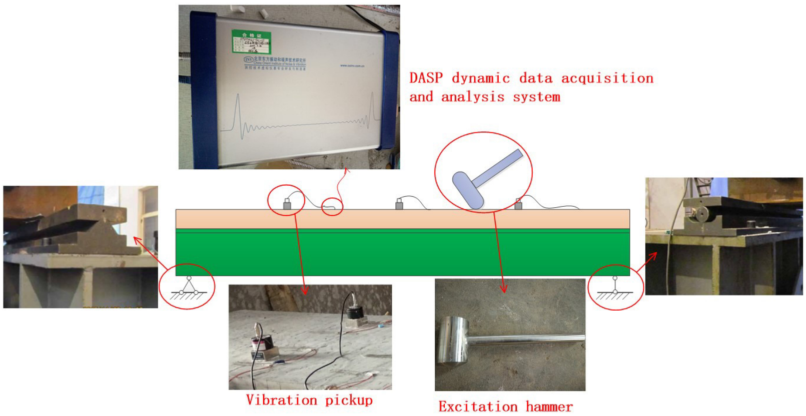


Figure 15 Test arrangement

1. At the initial stage of test specimen loading, the bond between the bottom composite beam and the top concrete slab is strong. The interface friction coefficient corresponding to this state is ∞.
2. When the test beam is loaded to about 70%~80% of the elastic ultimate load, the bond between interfaces is partially destroyed, corresponding to the interface friction coefficient $0<\mu<\infty$.
3. The load continued to increase until the test beam entered the plastic stage, and the interface between the bottom composite beam and top concrete slab was vertically stripped, and the bond between the interfaces was almost completely destroyed. The interface friction coefficient corresponding to this state is $\mu=0$.

The test results are shown in Table 2.

The formula(6) displays that the greater the friction coefficient between the steel and concrete surfaces, the greater the stiffness ratio α($\alpha=B_d/B_c$) between the contact region and the separation region, and therefore the greater the impact on the natural vibration frequency.

The first-order natural frequency of the test beam under different interface states is calculated, and the calculated result is compared with the measured results, as show in Table 2. In which, μ=0.7, 2.0 correspond to bond between interfaces is partially destroyed, *ie*, 0<μ<∞.

Table 2 The first order natural vibration frequency of single-stretched test beam (Hz)

μ	Calculated values	Test values
0	22.9812	22.9835
0.7	22.9819	23.0013
2.0	23.4150	
∞	32.1758	31.2825

In Table 2, the calculated results and test results are in good agreement, which indicate that it is feasible to use matrix transfer method to calculate the natural vibration frequency of laminated beams.

Taking the friction coefficient of interface between top concrete slab and bottom composite beam as 0.7, the first four order natural vibration frequencies of the test beam vibrating freely in two forms of equal wavelength and equal stiffness are calculated as shown in Table 3.

Table 3 Natural vibration frequency of single-stretched beam (Hz)

	Regardless of friction	Considering friction (equal wavelength)	Considering friction (equal stiffness)	Test values
1	22.9812	22.9812(22.9819)	22.9812(22.9819)	23.0013
2	118.0474	118.0482(118.0491)	118.0482(118.0495)	——
3	199.5153	199.5186(199.5195)	199.5186(199.5195)	——
4	221.4489	221.4520(221.4524)	221.4520(221.4529)	——

Note: the upper limit of natural frequencies considering friction effect is in parentheses.

The calculation results in Table 3 show that with the increase of the order, the influence of the interface friction on the natural vibration frequency of the beam is gradually significant. When the beam vibrates with equal wavelength and equal stiffness, the frequency difference is not obvious, less than 0.1%, which can be calculated in equal wavelength form.

5 CONCLUSION

In this paper, the laminated beam with uniform vertical displacement is taken as the research object, the calculate formula of section stiffness considering the action of friction and friction moment between the laminated interface is derived. Due to the stiffness of the contact zone and separation zone is different, when the beam vibrates freely in equal wavelength and equal stiffness forms respectively, the vibration mode distribution of the beam is different. The formulas for calculating natural frequencies of simply supported laminated beams and cantilever laminated beams are deduced by using transfer matrix method. Finally, the applicability of the formulas is verified by a test example of steel-concrete composite-laminated beam. The main conclusions are as follows:

(1) The calculation formula of section stiffness considering friction between overlapping interfaces is derived. The effect of friction resistance moment on section stiffness is greater than that of friction. The stiffness of cross section is mainly related to the friction coefficient between laminated interface and the stiffness of the top and bottom beams.

(2) Based on the matrix transfer method, the formulas for calculating the vibration frequency of laminated beams in equal wavelength and equal stiffness forms are derived. And the natural frequency solutions of simply supported laminated beams and cantilever laminated beams are given.

(3) At different interface states, the calculated results of natural vibration frequency of composite-laminated beam are agree well with the measured results.

ACKNOWLEDGMENTS

The work is supported by Natural Science Foundation of Hebei Province (E2020210017), project of Scientific Research Start-up Foundation of Quzhou University of Zhejiang, China (No.BSYJ202121).

Author's Contributions: Writing - original draft, X Liang; Test technical support, B Chen; Review & editing, WR He.

Editor: Pablo Andrés Muñoz Rojas

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