www.lajss.org
Latin American Journal of Solids and Structures 2 (2005) 29–39

Rupture of framed structures, lumped damage mechanics and Internet

Richard Espinoza, Julio Flórez-López*, Nayive Jaramillo, María E. Marante, Adriana Quero and Lorena Suarez

Facultad de Ingeniería, Universidad de Los Andes, Mérida 5101, Venezuela

Abstract

This paper describes a general framework for the description of the rupture of framed structures called Lumped Damage Mechanics. This theory combines Fracture Mechanics, Damage Mechanics and the concept of plastic hinge. In the particular case of reinforced concrete frames, the main mechanism of deterioration is cracking of concrete under overloads. Cracking in a frame element is lumped at the plastic hinges. Cracking evolution is assumed to follow a generalized Griffith criterion. Thus, an expression of the energy release rate of a plastic hinge with cracking is obtained. The behavior of the damaged plastic hinge is described via the strain equivalence hypothesis. This model of damage for framed structures has been implemented in a finite element program specially conceived for this purpose. This program is a Web-based parallel FE code that at the moment only has one element, the one based on the Lumped Damage Theory. The system is also composed by a preprocessor and a postprocessor that can be accessed via any commercial browser. The preprocessor allows for the digitalization of RC frames. The file with the data of the frame is sent via web to the FE program and the results can be visualized by the postprocessor or by results files.

1 Introduction

The concept of frame is commonly used to represent many civil and industrial engineering structures such as buildings, bridges, cranes and so on. This kind of structures is often subjected to overloads, typically earthquakes, but also impacts and blasts that may severely damage the structure or even produce its collapse.

Lumped Damage Mechanics is a general framework for the modeling and the numerical simulation of this phenomenon. This theory combines Fracture Mechanics, Damage Mechanics, and the concept of plastic hinge. Lumped Damage Mechanics has been used to model the behavior of reinforced concrete Frames [2, 3, 5–8], and steel frames [4].

* Corresponding author E-mail: iflorez@ula.ve Received 24 November 2004
From *Recent Developments in the Modelling of Rupture in Solids Conference*, ed. A. Benallal & S.P.B. Proença.

2 Lumped damage mechanics

2.1 Flexibility matrix of a Reinforced Concrete frame member with cracking

Let us consider a planar frame as shown in Figure 1a. A frame member between the nodes i and j of the frame is isolated. The generalized stress and strain matrices of the frame member are given, respectively, by $\mathbf{M}^t = (m_i, m_j, n)$ and $\mathbf{\Phi}^t = (\phi_i, \phi_j, \delta)$ where the terms m_i and m_j represent the flexural moments at the ends of the element, n is the axial force, ϕ_i and ϕ_j are the relative rotations of the frame member with respect to the chord and δ is the chord elongation (see Figures 1b and 1c).

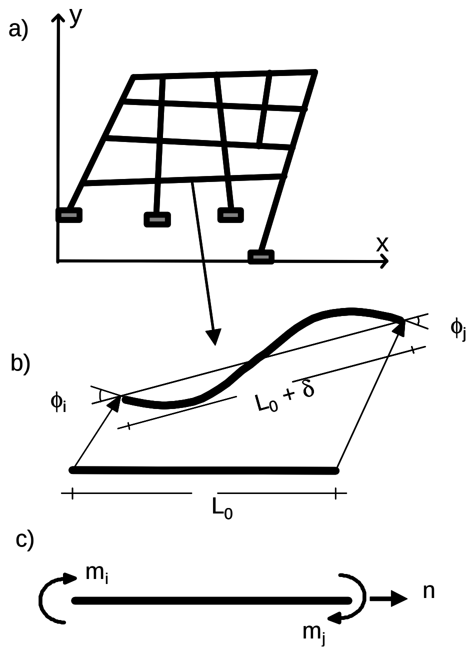


Figure 1: a) Planar Frame b) Generalized strains c) Generalized stresses.

Two main inelastic phenomena occur in a reinforced concrete structure subjected to overloads: yield of the reinforcement and concrete cracking. In order to model these phenomena in large and complex framed structures, the lumped dissipation representation of a frame member is usually adopted. This model consists in assuming that all inelastic phenomena can be lumped at special locations called plastic or inelastic hinges. Therefore a frame member is considered as the assemblage of an elastic beam-column and two inelastic hinges as shown in Figure 2.

The conventional theory of elastic-plastic frames is obtained by the introduction of a new internal variable that will be denoted generalized plastic strains $\mathbf{\Phi}_p^t = (\phi_i^p, \phi_j^p, 0)$, where the symbols ϕ_i^p y ϕ_j^p represent the plastic rotations of the inelastic hinges at the ends i and j.

Lumped Damage Mechanics is obtained by the introduction of a new set of hinge-related

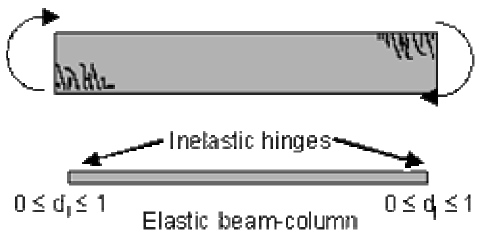


Figure 2: Lumped dissipation model and representation of cracking through damage variables.

internal parameters. This variable, denoted damage, measures the crack density in the element as indicated in Figure 2. Thus, the damage matrix is given by: $\mathbf{D} = (d_i, d_j)$, where d_i and d_j are damage parameters that can take values between zero (no cracking) and one (total damage). They represent cracking as lumped in the hinges i and j.

It can be noticed that the damage parameters do not measure crack length but crack density as the continuum damage variables. On the other hand, these variables are related to macroscopic cracks, as in the case of fracture mechanics, instead of microcracking density as the damage mechanics variable.

In the reference [3], the following elasticity law of a damaged reinforced concrete element was proposed:

$$\mathbf{\Phi} - \mathbf{\Phi_p} = \mathbf{F}(\mathbf{D})\mathbf{M} \tag{1}$$

where $\mathbf{F}$ is the flexibility matrix of a cracked frame member that is a function of the damage variable and has the following expression:

$$\mathbf{F}(\mathbf{D}) = \begin{bmatrix} \dfrac{F_{11}^0}{1-d_i} & F_{12}^0 & 0 \\ F_{21}^0 & \dfrac{F_{22}^0}{1-d_j} & 0 \\ 0 & 0 & F_{33}^0 \end{bmatrix} \tag{2}$$

The terms F_{ij}^0 represent the coefficients of the elastic flexibility matrix as given in textbooks of structural analysis.

It can be noticed that for damage values equal to zero, $\mathbf{F}$ becomes the elastic flexibility matrix of the classic structural analysis theory. If a damage variable tends to one, the corresponding flexibility term tends to infinite (or the stiffness tends to zero) and the inelastic hinge behaves as the internal hinges of the classic frame analysis. It is assumed that the damage parameters evolve continuously from zero to one following the generalized Griffith criterion that will be defined in the next section. In this way, stiffness degradation is represented by the model.

2.2 Generalized Griffith criterion

The complementary strain energy of a cracked frame member can be obtained from (1):

$$\mathbf{W}^* = \frac{1}{2}\mathbf{M}^t(\mathbf{\Phi} - \mathbf{\Phi}^p) = \frac{1}{2}\mathbf{M}^t\mathbf{F}(\mathbf{D})\mathbf{M} \tag{3}$$

Then, the energy release rates of the plastic hinges are given by:

$$\begin{aligned} G_i &= -\frac{\partial W^*}{\partial d_i} = \frac{m_i^2 F_{11}^0}{2(1-d_i)^2} \\ G_j &= -\frac{\partial W^*}{\partial d_j} = \frac{m_j^2 F_{22}^0}{2(1-d_j)^2} \end{aligned} \tag{4}$$

Therefore a generalized Griffith criterion for the inelastic hinge i can be defined in the following terms: there will be damage evolution (i.e. crack propagation) in the plastic hinge i only if the energy release rate G_i reaches the value of the crack resistance of the hinge:

$$\begin{cases} \dot{d}_i = 0 \text{ if } G_i - R(d_i) < 0 \text{ or } \dot{G}_i - \dot{R}(d_i) < 0 \\ \dot{d}_i > 0 \text{ if } G_i - R(d_i) = 0 \text{ and } \dot{G}_i - \dot{R}(d_i) = 0 \end{cases} \tag{5}$$

where R is crack resistance of the plastic hinge i. This function has been identified from experimental results and can be considered as a member-dependent property [2].

2.3 Plastic behavior of a frame member with cracking

Following [3], the behavior of a plastic hinge with cracking can be obtained by using the equivalent stress hypothesis. In continuum damage mechanics and poro-elasticity, this hypothesis states that the behavior of a damaged material can be expressed by the same relations of the intact material if the stress is substituted by the effective stress. By analogy with Continuum Damage Mechanics, the effective moment $\bar{m}_i$ on a plastic hinge is defined as:

$$\bar{m}_i = \frac{m_i}{1-d_i} \tag{6}$$

Thus the yield function of a plastic hinge with damage and kinematic hardening is given by:

$$f_i^y = \left| \frac{m_i}{1-d_i} - c\phi_i^p \right| - k_y \leq 0 \tag{7}$$

where c and k_y are member dependent parameters. The elasticity law (1), the Griffith criterion (5) and the yield function (7) define the constitutive law of a frame member with cracking and yielding. A procedure for the determination of the parameters of the model as a function of the properties of the materials and the frame element is described in [7].

2.4 Other Lumped Damage Mechanics concepts

Further extensions of the model described in the precedent sections have been proposed and were implemented in the portal. The most important of them is the concept of unilateral damage. In a frame member subjected to loadings that change sign, two different set of cracks appears in the frame member, one due to positive moments (positive cracks) and another as a consequence of negative moments (negative cracks). Within the framework of the Lumped Damage Mechanics, this effect can be represented by using two sets of damage variables instead of one: $\mathbf{D}^+ = (d_i^+, d_j^+)$ and $\mathbf{D}^- = (d_i^-, d_j^-)$, where the positive superscript indicates damage due to positive or negative moments (see Figure 3). The behavior of the frame element under this conditions presents a behavior that can described as "unilateral". This adjective, that comes from the Continuum Damage Mechanics, indicates that for positive moments, the negative cracks tend to close and have little or no influence on the behavior of the member and vice versa. A perfect unilateral behavior can be described by the following modification of the elasticity law (1):

$$\mathbf{\Phi} - \mathbf{\Phi_p} = \mathbf{F}(\mathbf{D}^+) < \mathbf{M} >_+ + \mathbf{F}(\mathbf{D}^-) < \mathbf{M} >_- \tag{8}$$

where the symbols $< x >_+$ and $< x >_-$ indicate, respectively, the positive and negative part of the variable x. i.e. $< x >_+ = x$ if $x > 0$; $< x >_+ = 0$ otherwise. $< x >_- = x$ if $x < 0$; $< x >_-$ $= 0$ otherwise. It can be noticed that positive damage has no influence at all in the compliance of the structure if the moments are positive and vice versa.

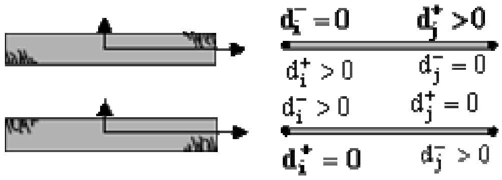


Figure 3: Unilateral damage

Two different energy release rates per hinge are now defined: G_i^+ and G_i^-. They are defined by an expression similar to (3) except that the moment is substituted by the positive part or the negative part of the moment and the damage for the corresponding positive or negative variable. The yield function can also be modified by the definition of an effective moment with the positive damage variable if positive cracks are active and vice versa (see [7], for additional details).

Another extension of the model consists in the use of modified forms of the Griffith criterion, in particular the one described in [8], was included in the portal in order to describe low cycle fatigue effects.

3 The portal of damage

As an attempt to attain an extensive use of the model just described, a special finite element program has been developed. This FE code will not be distributed amongst the users. Instead, the program can be accessed via Web at the following address *http://portalofdamage /cecalc.ula.ve/pdp/* (Figure 4). The portal is, at the time when this paper was written, in a stage of trial and debugging.

The user of the system that is connected for the first time to the portal is requested to create an account. In the subsequent visits to the site, the user will find the files of its previous and present applications saved in her/his account. The Account Manager, one of the components of the system is in charge of these functions.

Figure 4: Portal of Damage

In order to create the data for a new application or modify a previous one, the user employs a pre-processor that can be accessed via the portal (Figure 5). The semi-graphic Preprocessor is the second component of the system. The user is requested to describe the geometry of the structure: number of nodes and its coordinates, number of elements and its nodes and the properties of the member's cross section. These properties are the dimensions of the cross sections, the number of bars of the longitudinal and transversal reinforcement, their diameters, and their position in the cross section and along the frame element. Additionally, the user is requested to describe the uniaxial behavior of the concrete and the reinforcement. Finally the external loading, and imposed displacements as well as the proposed time steps must be supplied. The Preprocessor computes interaction diagrams for each inelastic hinge of the frame from this information and generates the input file for the analysis.

The third module of the system is the Job Manager. With this program the user chooses a data file from her/his account and run the FE program. The Job Manager is also used to monitor the execution.

The FE program called "Damage Processor" is the forth element of the system. This is a parallel non-linear Finite Element program based on the lumped damage model just described. The Damage Processor is summoned by the Job Manager and submits the computer replica of the structure to the overloads selected by the user. In the present state of the program, they can be quasi-static or earthquake loadings. Geometrically nonlinear effects can also be taken into account.

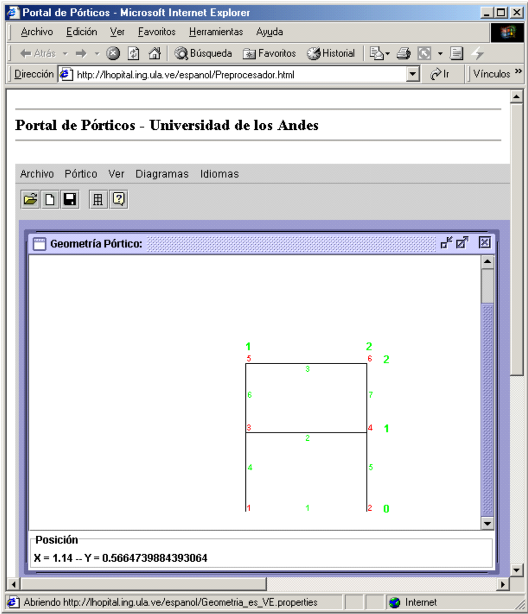


Figure 5: Preprocessor

The user has access to the results of this analysis by a Postprocessor that can also be accessed via commercial browsers (Figure 6). The Postprocessor is the fifth and last component of the system. With the Postprocessor, the user can obtain variable vs. variable and variable vs. time graphs as well as spatial distributions of damage and other hinge related variables at any instant.

4 Numerical examples

In this section two numerical simulations carried out with the portal are shown. The first example is a RC column in cantilever subjected in laboratory to a complex loading program that represents the actions that a building column must withstand during an earthquake. They include cyclic lateral displacements and variable axial forces [1]. The experimental behavior is represented in the graph of lateral displacements vs. lateral forces shown in Figure 7a. The numerical simulation obtained with the portal is shown in Figure 7b.

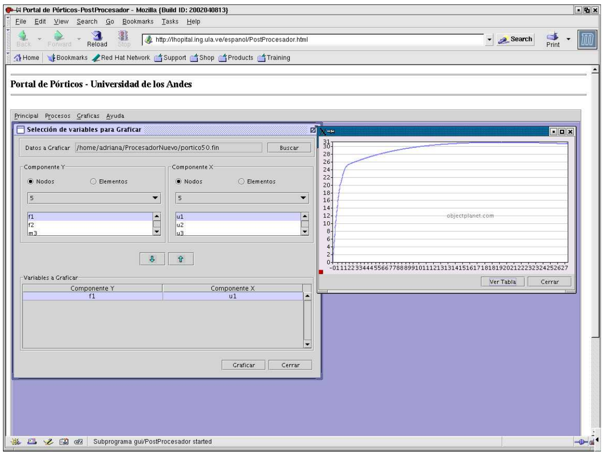
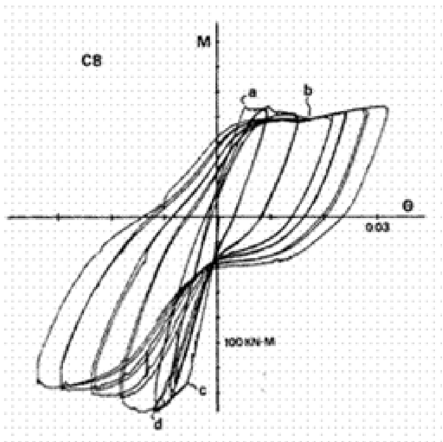
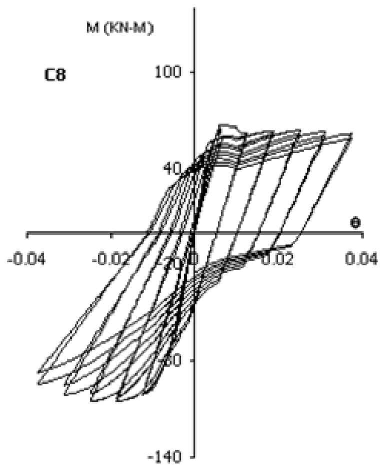


Figure 6: Postprocessor



(a)



(b)

Figure 7: RC column subjected to lateral displacements and variable axial forces (a) Experimental results [1] (b) Numerical simulation.

The second and last example is a two-story RC frame that was subjected in laboratory to constant axial forces and to an imposed lateral displacement at the top of the frame as shown in Figure 8 (see [9]). The experimental results are shown in Figure 9 in a graph of lateral displacement vs. lateral force. The results of the numerical simulation with the portal are shown in Figures 10 and 11. In Figure 10, the same graph of lateral displacement vs. lateral force is represented. Figure 11 shows the damage distribution at the end of the simulation.

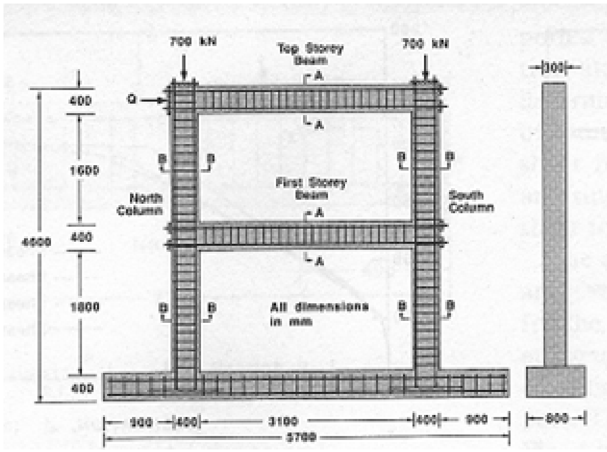


Figure 8: RC frame [9]

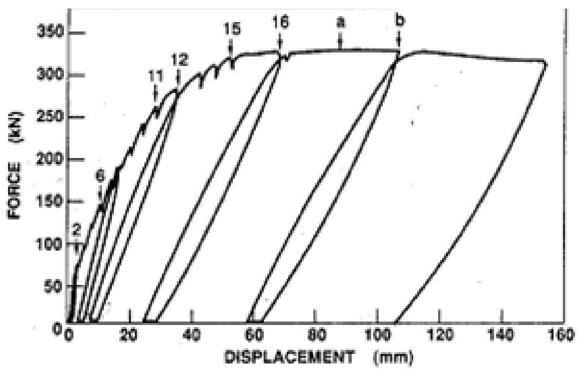


Figure 9: Lateral displacement vs. lateral force [9]

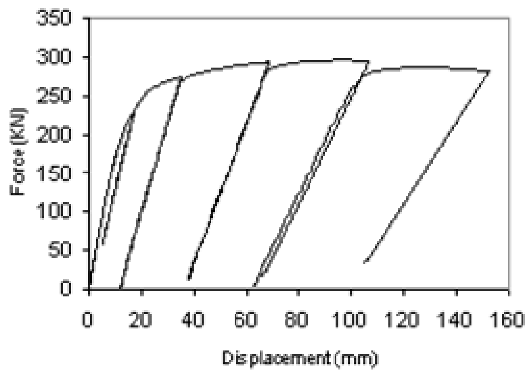


Figure 10: Lateral displacement vs. lateral force, numerical simulation

5 Final remarks

Hundred of thousands of buildings and other civil engineering structures around the world are vulnerable to the action of severe earthquakes, and these earthquakes will eventually occur. This problem is especially acute in less developed countries. Web-based structural analysis programs, as the one described here, are promising tools for the evaluation, diagnosis and the design of retrofitting projects of these structures at a large scale. The potential advantages in economy and author's rights protection are evident and do not need to be emphasized.

The system that has been described is in experimental stage and there are a number of problems that must be solved: automatic convergence schemes, robustness and velocity.

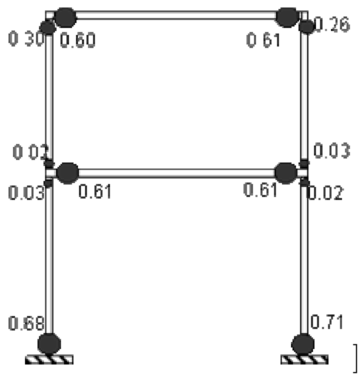


Figure 11: Damage distribution at the end of the simulation.

References

[1] D. P. Abrams. Influence of axial force variations on flexural behavior of reinforced concrete columns. *ACI Struc. J.*, May-June (1987).

[2] A. Cipollina, A. López-Inojosa, and J. Flórez-López. A simplified damage mechanics approach to nonlinear analysis of frames. *Computers & Structures*, 54(6):1113–1126, 1995.

[3] J. Flórez-López. Frame analysis and continuum damage mechanics. *J. Eur. Mech.*, 17(2):269–284, 1998.

[4] P. Inglessis, S. Medina, A. López, R. Febres, and J. Flórez-López. Modeling of local buckling in tubular steel frames by using plastic hinges with damage. *Steel & Composite Structures*, 2(1):21–34, 2002.

[5] M. S. lvares, S. P. B. Proena, and R. Billardon. Estudo e emprego de um modelo de dano localizado: aplicação à vigas em concreto armado. In *Proc. XX CILAMCE*, 1999.

[6] M. E. Marante and J. Flórez-López. Three-dimensional analysis of reinforced concrete frames based on lumped damage mechanics. *Int. J. Solids Struct. (in press).*

[7] M. E. Perdomo, A. Ramirez, and J. Flórez-López. Simulation of damage in rc frames with variable axial forces earthquake. *Engineering & Structural Dynamics*, 28(3):311–328, 1999.

[8] E. Thomson, A. Bendito, and J. Flórez-López. Simplified model of low cycle fatigue for rc frames. *Journal of Structural Engineering ASCE*, 124(9):1082–1086, 1998.

[9] F. J. Vechio and M. B. Emara. Shear deformation in reinforced concrete frames. *ACI Struc. J.*, 89(1), 1992.