Latin American Journal of
Solids and Structures
www.lajss.org

# Dynamical stability and parametrical vibrations of the laminated plates with complex shape

**Abstract**

The problem of nonlinear vibrations and stability analysis for the symmetric laminated plates with complex shape, loaded by static or periodic load in-plane is considered. In general case research of stability and parametric vibrations is connected with many mathematical difficulties. For this reason we propose approach based on application of R-functions theory and variational methods (RFM).The developed method takes into account pre-buckle stress state of the plate. The proposed approach is demonstrated on testing problems and applied to laminated plates with cutouts. The effects of geometrical parameters, load, boundary conditions on stability regions and nonlinear vibrations are investigated.

**Keywords**

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## 1 INTRODUCTION

Laminated composite plates are frequently used in various engineering applications in the aviation, aerospace, marine, mechanical and others industries. The use of composite materials requires complex analytical and numerical methods in order to predict accurately their response to external loading. A recent review of current developments in non-linear vibration analysis and dynamic stability of the laminated plates and shells has been presented in references [6,9,11,13,14], ets. From presented review it follows that buckling problems with non-uniform pre-buckle stress state are of special interest. There are few works in which vibration, buckling and parametric instability behavior of a laminated plate with internal cutouts were studied [10,12]ets. It should be noted that the finite element method (FEM) remains the only way for dealing with complex structures and the most versatile.

In this work we propose alternative to FEM approach based on using variational methods and R-functions theory (RFM). The developed method takes into account pre-buckle stress-state and allows investigating the laminated plates of an arbitrary form, in particular case with internal free, simply supported and clamped cutouts. Formerly this approach was developed in references [2,3] for isotropic and orthotropic plates. The aim of this paper is extending the theoretical model of this method in order to carry out the nonlinear analysis of laminated plates with an arbitrary shape and different boundary conditions. The dynamic instability and buckling characteristics of the laminated plates with cutouts are discussed.

## 2 PROBLEM FORMULATION

Geometrically nonlinear vibration of simmetricaly laminated composite plates subjected to in-plane compressive periodic edge loading $p = p_0 + p_t \cos\theta t$ is studied. It is assumed that the delamination of the layers is absent. The mathematical formulation of the problem is made in the framework of the classical laminated plate theory. Neglecting rotatory inertia the equation of equilibrium [1] may be written as:

$$N_{11,x} + N_{12,y} = 0\,, \tag{1}$$

$$N_{12,x} + N_{22,y} = 0\,, \tag{2}$$

$$M_{11,xx} + 2M_{12,xy} + M_{22,yy} + N_{11}w_{,xx} + 2N_{12}w_{,xy} + N_{22}w_{,yy} = mw_{,tt}\,, \tag{3}$$

where $N_{11}, N_{22}, N_{12}$ and $M_{11}, M_{22}, M_{12}$ are stress resultants:

$$\vec{N} = \begin{pmatrix} N_{11} \\ N_{22} \\ N_{12} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{pmatrix},\ \vec{M} = \begin{pmatrix} M_{11} \\ M_{22} \\ M_{12} \end{pmatrix} = \begin{pmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{pmatrix} \begin{pmatrix} -w_{,xx} \\ -w_{,yy} \\ -2w_{,xy} \end{pmatrix}.$$

Deformations $\vec{\varepsilon} = \left(\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}\right)^T$ are expressed as [1]

$$\varepsilon_{11} = u_{,x} + \frac{1}{2}w_{,x}^2\,, \qquad \varepsilon_{22} = v_{,y} + \frac{1}{2}w_{,y}^2\,,\ \ \varepsilon_{12}^{\ L} = v_{,x} + u_{,y} + w_{,x}w_{,y}\,,$$

where u,v,w are displacements of the plate in directions Ox, Oy and Oz respectively.

For convenience we introduce the following notation

$$\vec{\varepsilon} = \vec{\varepsilon}^{\ L} + \vec{\varepsilon}^{\ N}\,,\ \vec{N} = \vec{N}^{\ L} + N^{\ N}\,,$$

where

$$\vec{\varepsilon}^{\ L} = \left(\varepsilon_{11}^{\ L}, \varepsilon_{22}^{\ L}, \varepsilon_{12}^{\ L}\right)^T,\ \vec{\varepsilon}^{\ N} = \left(\varepsilon_{11}^{\ N}, \varepsilon_{22}^{\ N}, \varepsilon_{12}^{\ N}\right)^T$$

$$\varepsilon_{11}^{L} = u_{,x}, \varepsilon_{22}^{L} = v_{,y}, \varepsilon_{12}^{L} = v_{,x} + u_{,y}, \ \varepsilon_{11}^{N} = \frac{1}{2}\left(w_{,x}\right)^{2}, \varepsilon_{22}^{N} = \frac{1}{2}\left(w_{,y}\right)^{2}, \varepsilon_{12}^{N} = w_{,x}w_{,y}.$$

Then stress resultants $\vec{N}^{L} = \left(N_{11}^{L}, N_{22}^{L}, N_{12}^{L}\right)^{T}$ and $\vec{N}^{N} = \left(N_{11}^{N}, N_{22}^{N}, N_{12}^{N}\right)^{T}$ may be presented as follows

$$\vec{N}^{(L)} = C\varepsilon^{(L)}, \quad \vec{N}^{(N)} = C\varepsilon^{(N)}.$$

The components $C_{ij}, D_{ij}$ $\left(ij = 11, 22, 12, 16, 26, 66\right)$ of the stiffness matrices C and D are defined as[1]:

$$\left(C_{ij}, D_{ij}\right) = \sum_{s=1}^{N} \int_{h_s}^{h_{s+1}} B_{ij}^{\left(s\right)} \left(1, z^2\right) dz,$$

where $B_{ij}^{(s)}$ are mechanical characteristics of the s-layer.

The system of equations (1)-(3) is supplemented by corresponding boundary conditions.

## 3. METHOD OF SOLUTION

The proposed method is based on solving a number of auxiliary problems.

1. Solving problem about pre-buckle stress state of the laminated plate.
In order to determine the pre-buckle stress state of the plate let us consider the following system

$$\begin{aligned} N_{11,x}^{L} + N_{12,y}^{L} &= 0, \\ N_{12,x}^{L} + N_{22,y}^{L} &= 0, \end{aligned} \tag{4}$$

supplemented on the loaded part of the border $\partial\Omega_1$ by the following boundary conditions

$$N_{n}^{(L)}\left(u_1, v_1\right) = -1, \quad T_{n}^{(L)}\left(u_1, v_1\right) = 0, \quad \left(x, y\right) \in \partial\Omega_1 \tag{5}$$

Operators $N_{n}^{L}, T_{n}^{L}$ in (5) is defined by formulas

$$N_{n}^{L} = N_{11}^{L} l^2 + N_{22}^{L} m^2 + 2N_{12}^{L} lm, T_{n}^{L} = N_{12}^{L}\left(l^2 - m^2\right) + \left(N_{11}^{L} - N_{22}^{L}\right) lm,$$

where $l = \cos(\vec{n}, Ox)$, $m = \cos\left(\vec{n}, Oy\right)$, $\vec{n}$ is normal vector to border of the domain. Boundary conditions on the remain part depend on way of fixing.

Solution $u_1\left(x,y\right), v_1\left(x,y\right)$ of this system (4)-(5) may be found by RFM (R-functions method). Variational formulation of the problem (4)-(5) is reduced to finding minimum of the functional

$$I(u_1, v_1) = \frac{1}{2}\iint_{\Omega} (N_{11}^{(L)}\varepsilon_{11} + N_{22}^{(L)}\varepsilon_{22} + N_{12}^{(L)}\varepsilon_{12})d\Omega + \int_{\partial\Omega_1} N_n^{(L)}\left(u_1 l + v_1 m\right) d\Omega_1 \tag{6}$$

We seek the minimum of the functional (6) on the set of basis functions constructed by R-functions theory [7,8].

Forces $\vec{N}^{\,0} = \left(N_{11}^{\,0}, N_{22}^{\,0}, N_{12}^{\,0}\right)^T$ are defined after finding solutions $u_1$ and $v_1$,

$\overrightarrow{N}^{(0)}\left(u_1, v_1\right) = \mathbf{C} \cdot \overrightarrow{\varepsilon^{(0)}}\left(u_1, v_1\right)$,

where

$$\overrightarrow{\varepsilon^{(0)}}\left(u_1, v_1\right) = (\varepsilon_{11}^{(0)}, \varepsilon_{22}^{(0)}, \varepsilon_{12}^{(0)})^T, \quad \varepsilon_{11}^{(0)} = u_{1,x}, \; \varepsilon_{22}^{(L)} = v_{1,y}, \; \varepsilon_{12}^{(L)} = u_{1,y} + v_{1,x}.$$

2. Finding buckling load.

Let us find the critical load provided that compressive load is varied proportionally to some parameter $\lambda$, that is, from the following equation

$$L_{33}w = \lambda \cdot Nl_3\left(u, v, w\right). \tag{7}$$

The critical value of the parameter $\lambda$ is found by energy approach. Let us write the appropriate functional:

$$I(w) = \frac{1}{2}\iint_{\Omega} [(M_{11}\chi_{11} + M_{22}\chi_{22} + M_{12}\chi_{12}) + \lambda(N_{11}^{\,0}(w_{,x})^2 +$$
$$+ N_{22}^{\,0}(w_{,y})^2 + N_{12}^{\,0}\, w_{,x}w_{,y})]dxdy\,.$$

As before, minimization of the functional is performed on the set of basis functions, constructed using the RFM. As a result of the Ritz's method, this problem is reduced to the eigenvalue problem.

3. Solution linear vibration problem of the laminated plate subjected to in-plane compressive static edge loading $p_0$. Above mentioned problem is solved by RFM too. The corresponding functional is

$$I = U - V, \tag{8}$$

where U is total potential energy taking into account the influence of forces in the median plane:

$$U(w) = \frac{1}{2}\iint_{\Omega} \big[ M_{11}\chi_{11} + M_{22}\chi_{22} + M_{12}\chi_{12} +$$

$$+ p_0 \;\; N_{11}^{L}\,(u_1, v_1)(w_{,x})^2 + N_{22}^{L}\,(u_1, v_1)(w_{,y})^2 + N_{12}^{L}\,(u_1, v_1) w_{,x} w_{,y} \;\Big] d\Omega,$$

and $V$ is kinetic energy:

$$V = \frac{m_1 {\Omega_L}^2}{2}\iint_{\Omega} w^2 d\Omega \,.$$

Let us denote eigenfunction corresponding to the first mode of free linear vibrations of loaded plate by $w_1$, $\Omega_L$ is relative natural frequency.

4. Solutions of the auxiliary problem like elasticity problem. Let us consider the following system of the equations:

$$\begin{aligned} N_{11,x}^{L} + N_{12,y}^{L} &= -Nl_1 \;\; w_1 \;\;, \\ N_{12,x}^{L} + N_{22,y}^{L} &= -Nl_2 \;\; w_1 \end{aligned} \tag{9}$$

The right side of the system (9) has the following kind

$$Nl_1 \;\; w_1 \;\; = N_{11,x}^{N} + N_{12,y}^{N} \,, \qquad Nl_2 \;\; w_1 \;\; = N_{12,x}^{N} + N_{22,y}^{N}$$

This system is supplemented by the boundary conditions:

$$N_n^{(L)} = -N_n^{(N)} \;\; w_1 \;\;, \quad T_n^{(L)} = -T_n^{(N)} \;\; w_1 \;\;, \tag{10}$$

where

$$N_n^{\;N} \;\; = N_{11}^{\;N}\, l^2 + N_{22}^{\;N}\, m^2 + 2N_{12}^{\;N}\, lm \,, T_n^{\;N} \;\; = N_{12}^{\;N} \;\; l^2 - m^2 \;\; + \;\; N_{11}^{\;N} \;\; - N_{22}^{\;N} \;\; lm \,.$$

The solution of system (9)-(10) is found by RFM. Denote this solution by $u_2 \;\; x, y \;\; , v_2 \;\; x, y \;\;$.

Variational formulation of the problem is reduced to finding minimum of the following functional

$$I(u_2,v_2)=\frac{1}{2}\iint_{\Omega}(N_{11}^{L}\,\varepsilon_{11}+N_{22}^{L}\,\varepsilon_{22}+N_{12}^{L}\,\varepsilon_{12}-2(Nl_1(w_1)u_2+Nl_2(w_1)v_2))d\Omega+$$
$$+\int_{\partial\Omega}N_n^{(N)}(w_1)(u_2l+v_2m)+T_n^{(N)}(w_1)(-u_2m+v_2l)d\Omega_1\,.$$

5. Solution of non-linear vibration problem. Let us represent unknown functions $u,v,w$ in the following way

$$w\ x,y,t\ =y\ t\ \cdot w_1\ x,y\ ,$$
$$u\ x,y,t\ =pu_1\ x,y\ +y^2\ t\ \cdot u_2\ x,y\ ,\ v\ x,y,t\ =pv_1\ x,y\ +y^2\ t\ \cdot v_2\ x,y\ . \tag{11}$$

Substituting expressions (11) into equation (1)-(3), and using the Bubnov-Galerkin method, we receive ordinary differential equation:

- in case of static load
$$y''(t)+\Omega_L^2\ \ y(t)+\beta\cdot y^3(t)\ \ =0, \tag{12}$$
- in case of periodic load
$$y''(t)+\Omega_L^2\ \ (1-2k\cdot\cos(\theta t))y(t)+\beta\cdot y^3(t)\ \ =0. \tag{13}$$

Here $k$ and $\beta$ is defined by the following expressions:

$$k=\frac{p_t}{2p_{kr}},\ \ \beta=-\frac{\iint_{\Omega}\ N_{11}(u_2,v_2,w_1)\cdot w_{1,xx}+2N_{12}(u_2,v_2,w_1)\cdot w_{1,xy}+N_{22}(u_2,v_2,w_1)\cdot w_{1,yy}\ \ d\Omega}{m_1\Omega_L^2\left\|w_1\right\|^2}.$$

To solve the equation (12) let us present unknown function $y(t)$ in the form $y(t)=A\cos\omega_N t$. Using the Bubnov-Galerkin method, we receive the dependence between amplitude A and frequency ratio $\nu=\dfrac{\omega_N}{\Omega_L}$:

$$\nu=\sqrt{1+\frac{3}{4}\beta A^2}.$$

To identify areas dynamic instability, instead of (13), we use the equation Mathieu:

$$y_t''\ t\ +\Omega_L^2\ \ 1-2k\cdot\cos\ \ \theta t\ \ \ y\ \ t\ \ =0\,.$$

The main area of dynamic instability (near $\theta = 2\Omega_L$) is limited by values $\theta_1$ and $\theta_2$ [4]:

$$\theta_1 = 2\Omega_L\sqrt{1-k}\,,\quad \theta_2 = 2\Omega_L\sqrt{1+k}\,.$$

For the analysis of nonlinear vibrations after the loss of stability we use nonlinear equation (13). Dependence between the frequency ratio $\theta / 2\Omega_L$ and amplitude A are determined as follows [4]:

$$A = \frac{2}{\sqrt{3\beta}}\sqrt{\frac{\theta^2}{4\Omega_L^2} - 1 \pm k}.$$

## 4. NUMERICAL RESULTS

To illustrate our approach let us consider symmetrically laminated plate with central cutout (Fig. 1). In this case it is needed to determine the pre-buckle stress state of the laminated plate. Suppose the plate is loaded longitudinally along the edges parallel to axis OY ($x = \pm\frac{a}{2}$).

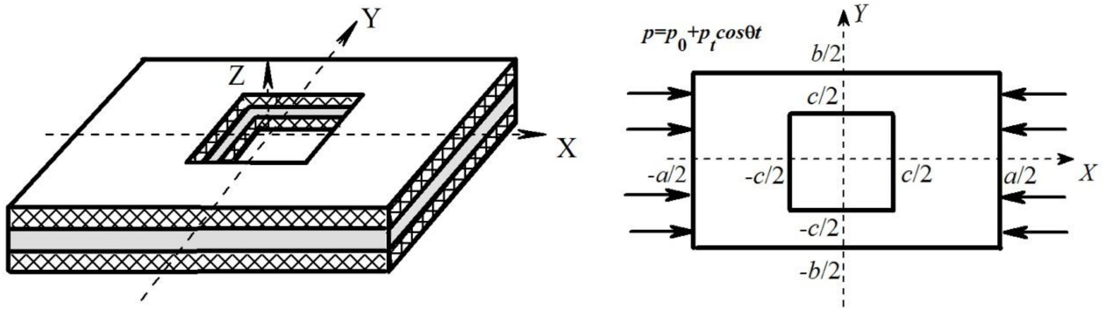



Figure 1 Form of plate with cutout.

Boundary conditions of two types are considered:

1. The plate is assumed simply supported on the outer border, but cutout is free (SS-F):

$$\begin{gathered} w = 0,\ M_x = 0,\ N_x = -p,\ N_{xy} = 0,\quad x,y \in \partial\Omega_1,\ \partial\Omega_1 : x = \pm\frac{a}{2}; \\ w = 0, M_y = 0,\ u = 0,\ N_y = 0,\quad x,y \in \partial\Omega_2,\ \partial\Omega_2 : y = \pm\frac{b}{2}; \\ M_n = 0,\ Q_n = 0,\ N_n = 0,\ T_n = 0,\quad x,y \in \partial\Omega_3,\ \partial\Omega_3 = \partial\Omega \setminus \partial\Omega_1 \cup \partial\Omega_2\ . \end{gathered} \tag{14}$$

2. The plate is simply supported on the all border (SS-SS):

$$
\begin{aligned}
&w = 0,\ M_x = 0,\ N_x = -p,\ N_{xy} = 0,\quad x,y \in \partial\Omega_1,\ \partial\Omega_1 : x = \pm\frac{a}{2};\\
&w = 0,\ M_n = 0,\ N_n = 0,\ T_n = 0,\quad x,y \in \partial\Omega_4,\ \partial\Omega_4 = \partial\Omega \setminus \partial\Omega_1 .
\end{aligned}
\tag{15}
$$

Numerical results are obtained for material with the following relations for the elasticity coefficients: E11=141.0 Gpa, E22=9.23 Gpa, G12=G13=5.95 Gpa, G23=2.96 Gpa, ν12= 0.313.
The structures of solution [7,8] for above mentioned boundary value problems (4)-(5), (7), (8), (9)-(10) are taken in the form:

1. boundary conditions SS-F (14):

$$w_1 = \omega_0 \cdot P_0 ; u_i = \omega_2 \cdot P_i,\ v_i = P_{i+2},\ i = 1,2 ;$$

2. boundary conditions SS-SS (15):

$$w_1 = \omega \cdot P_0 ; u_i = P_i,\ v_i = P_{i+2},\ i = 1,2 ,$$

where $P_0, P_j \ \ j = \overline{1,4}$ are indefinite components of the structure presented as an expansion in a series of some complete system (power polynomials, trigonometric polynomials, splines etc.), $\omega\ x,y\ = 0, \omega_0\ x,y\ = 0, \omega_2\ x,y\ = 0$ are equations of the boundary domain $\partial\Omega$ and its parts $\partial\Omega_0 = \partial\Omega_1 \cup \partial\Omega_2$ and $\partial\Omega_2$. For constructions of functions $\omega\ x,y\ , \omega_0\ x,y\ , \omega_2\ x,y$ theory of the R-functions is used:

$$\omega\ x,y\ =\ f_1 \wedge_0 f_2\ \wedge_0\ f_3 \vee_0 f_4\ ,\ \omega_0\ x,y\ = f_1 \wedge_0 f_2,\ \omega_2\ x,y\ = f_2 \tag{16}$$

where $\wedge_0, \vee_0$ are R-operations [7,8] represented below

$$x \wedge_0 y = x + y - \sqrt{x^2 + y^2},\ x \vee_0 y = x + y + \sqrt{x^2 + y^2} .$$

Functions $f_i, i = 1..4$ in (16) are determined as

$$f_1 = \frac{1}{a}\left(\left(\frac{a}{2}\right)^2 - x^2\right) \geq 0,\ f_2 = \frac{1}{b}\left(\left(\frac{b}{2}\right)^2 - y^2\right) \geq 0,\ f_3 = \frac{1}{c}\left(x^2 - \left(\frac{c}{2}\right)^2\right) \geq 0,\ f_4 = \frac{1}{c}\left(y^2 - \left(\frac{c}{2}\right)^2\right) \geq 0$$

To validate the proposed approach we investigate cross-ply four layers laminated plate with free cutout, boundary conditions (14) and for a=b=0.5m, h=0.005m. This problem had been solved by Dash S. and others in [5]. The accuracy and the efficiency of the present method are established through comparison of non-dimensional buckling load $\overline{\lambda} = \frac{N_x b^2}{E_2 h^3}$ with [5], fig. 2.

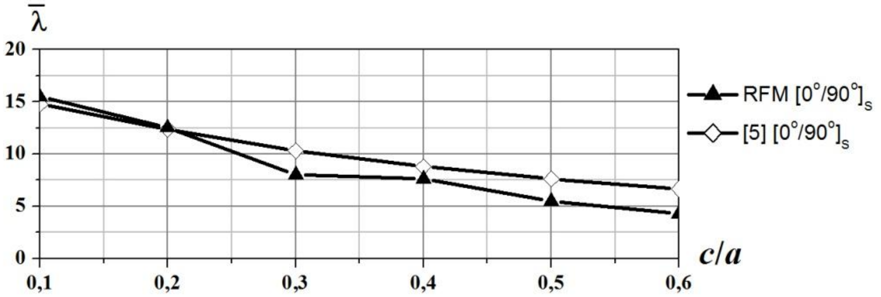


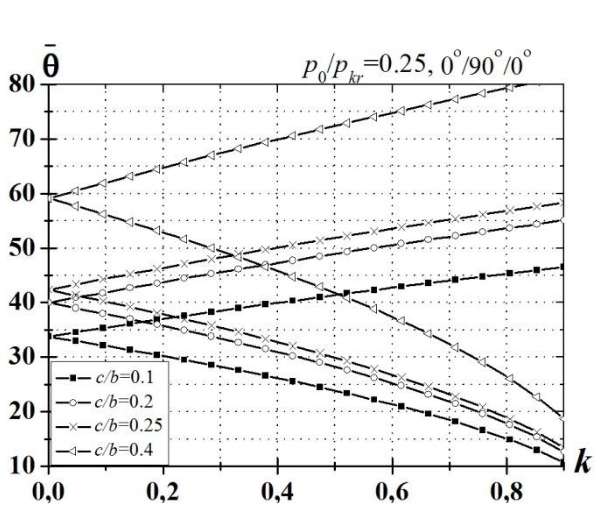

Figure 2 Non-dimensional buckling load $\overline{\lambda} = \frac{N_x b^2}{E_2 h^3}$ with different ratios of cutout.

Further calculations are carried out for geometrical parameters a/b=2, h=0.005m and boundary conditions (15). The influence of cutout size on non-dimensional frequencies $\Lambda = \Omega_L a^2 \sqrt{\rho / E_2 h^2}$ for various values of static component of load $p_0$ is presented in the table 1.

Table 1. Effect of cutout size on natural frequency $\Lambda = \Omega_L a^2 \sqrt{\rho / E_2 h^2}$

| $c / b$ | $0^0 / 90^0 / 0^0$ | | | $0^0 / 90^0 / 90^0 / 0^0$ | | | $0^0 / 90^0 / 0^0 / 90^0 / 0^0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $p_0 / p_{kr}$ | | | | | | | | |
| | 0 | 0.25 | 0.75 | 0 | 0.25 | 0.75 | 0 | 0.25 | 0.75 |
| 0.1 | 19.197 | 16.894 | 10.051 | 19.290 | 16.886 | 9.923 | 19.038 | 16.605 | 9.733 |
| 0.2 | 22.937 | 19.979 | 11.684 | 22.533 | 19.602 | 11.453 | 22.082 | 19.194 | 11.185 |
| 0.25 | 24.237 | 21.169 | 12.535 | 23.825 | 20.842 | 12.556 | 23.354 | 20.388 | 12.080 |
| 0.4 | 32.835 | 29.564 | 20.527 | 31.874 | 28.546 | 19.504 | 30.879 | 27.447 | 17.993 |

Effect of parameter of cutout size ($c / b = 0.1, 0.2, 0.25, 0.4$) on instability regions ($\overline{\theta}_1 = 2\lambda\sqrt{1-k}$, $\overline{\theta}_2 = 2\lambda\sqrt{1+k}$) are studied for $p_0 / p_{kr}$ =0.25, 0.75, fig. 3. Obtained results demonstrate the same behavior in both of the cases: for a larger value of $c / a$ the loss of instability occurs at lager values of excitation frequency $\vartheta$ and response zones occupy greater area. It should also be noted that instability zones for $p_0 / p_{kr}$ =0.25 correspond to greater value of excitation frequency $\vartheta$.



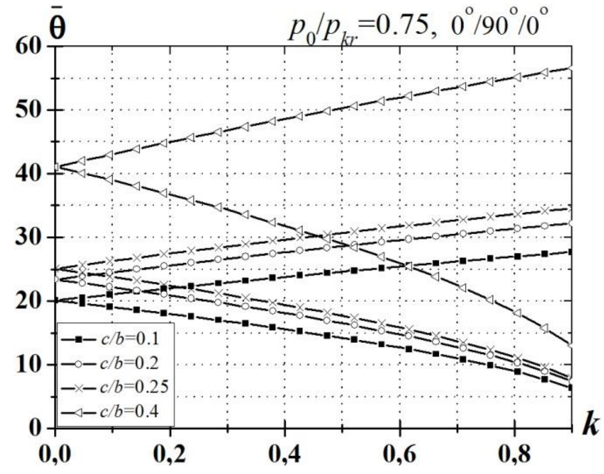




Figure 3. Instability zones for different size of cutout (SS-SS, $p_0 / p_{kr}$ =0.25,0.75).

Instability analysis is fulfilled for 3,4,5 layers plates. (fig.4). Results are obtained for $p_0 / p_{kr}$ =0.25 and $c / b = 0.2, 0.4$ . More significantly the number of layers effects on instability domains for plates with large cutouts.

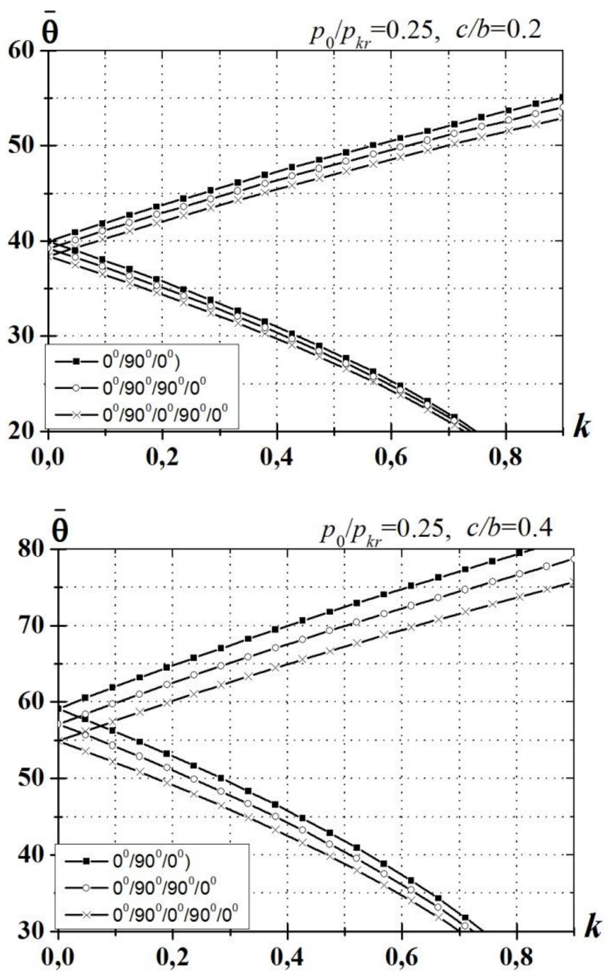



Figure 4. Instability zones for different number of layers (SS-SS, $p_0 \,/\, p_{kr}$ =0.25, $c \,/\, b\, =\, 0.2{,}0.4$ ).

Nonlinear parametric vibrations are investigated for 3-layers plate. Effect of load parameter $p_t$ on response curves is shown on fig.5. Calculations are performed for $c \,/\, b = 0.1{,}0.2$, $p_0 \,/\, p_{kr}$ $=$0.25. Decreasing of parameter $p_t$ leads to a convergence of curves.

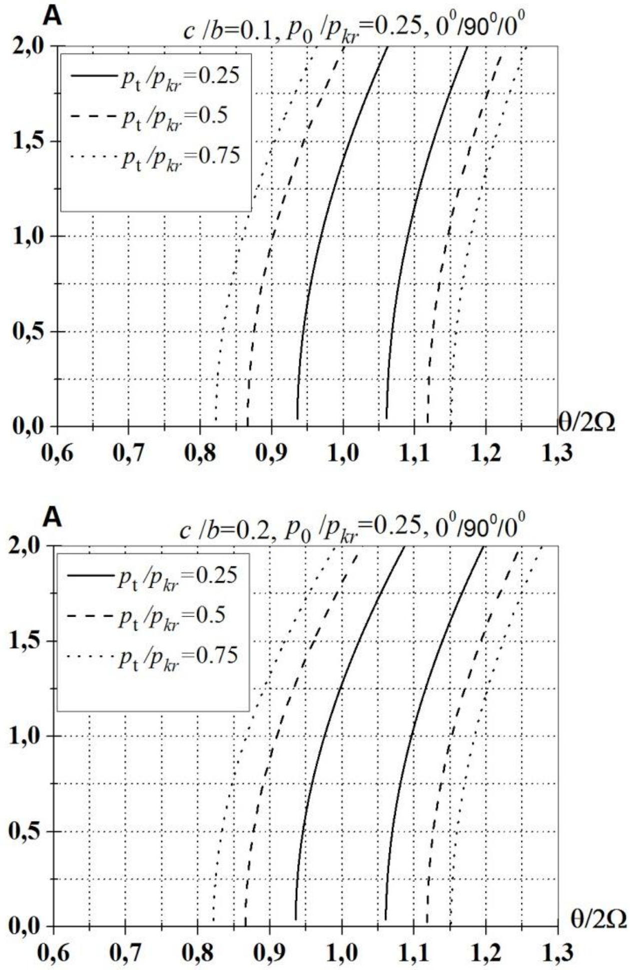



Figure 5. Response curve for different values of parameter $p_t$ (SS-SS, $p_0 / p_{kr}$ =0.25, $c / b = 0.1, 0.2$).

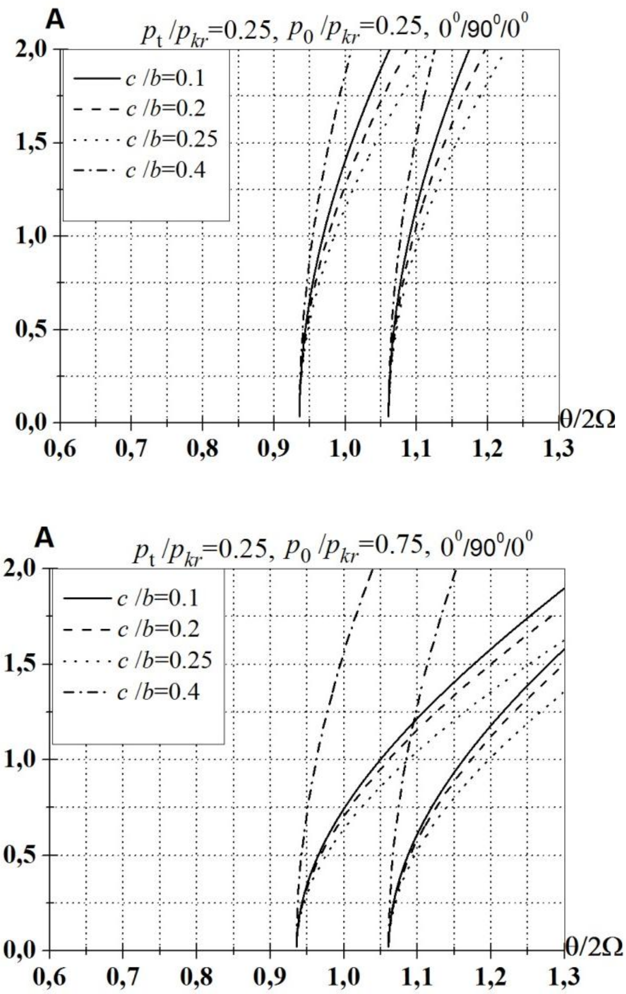



Figure 6. Response curve for different size of cutout (SS-SS, $p_0 / p_{kr}$ =0.25, 0.75).

The effect of size of cutout on the response curves is analyzed for $p_0 / p_{kr}$ =0.25, 0.75, $p_t / p_{kr}$ =0.25. The extension of cutout leads to increase of the vibration amplitude. Change of the static component of the load $p_0$ affects the slope of response curves.

## 5 CONCLUSIONS

An effective method to investigate dynamical stability and nonlinear vibrations of symmetrically laminated plates with a complex form is developed. The proposed method is based on the original meshless discretization procedure in the time and variational methods combined with R-functions theory. Due to application of the developed approach the initial nonlinear problem is reduced to sequence of auxiliary linear problems and nonlinear ordinary differential equations (ODEs) with respect to time. The present approach has advantage of being suitable for considering of different types of the boundary conditions in domains of arbitrary shape. The proposed method has been applied to study dynamical stability and nonlinear vibrations of simply supported rectangular plate with both free and simply supported central square cutout. The numerical results for buckling loads, instability regions and amplitude-frequency response curves are obtained for various size of cutout, number of layers, parameters of load. Note that due to increase of cutout size of simply supported plate, instability regions tend to shift to higher excitation frequency with extension of instability regions, showing destabilizing effect of cutout on behavior of plate. The effect of layers number on instability zones is not significant for plates with small cutout, but increase of cutout leads to essential influence of layers number. The amplitude-frequency response curves change the slope and location according to variation of values of the load components.

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