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Thermal effect on free vibration analysis of functionally graded arbitrary straight-sided plates with different cutouts

Abstract

In this paper, free vibration of functionally graded nonuniform straight-sided plates with circular and non-circular cutouts has been investigated. Moreover, thermal effects on free vibration analysis and the effects of various parameters on natural frequencies of these plates were evaluated. The material properties were assumed to be graded across thickness, which vary according to the linear distribution law. The investigated parameters in this study are: (1) cutout size (2) type of loading (3) different boundary conditions. It should be mentioned that the obtained results of thermal effect on free vibration of the FG nonuniform straight-sided plates (such as skew and trapezoidal plates) with cutouts have not been studied yet. Therefore, the results of this investigation can be implemented in future studies.

Keywords

functionally graded materials, square/skew/trapezoidal plates, circular/non-circular cutouts, thermal environment

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1 INTRODUCTION

Structural components such as turbines, aircraft engines and space vehicles are often exposed to very high temperatures and thermal shocks that inevitably induce several thermal stresses cause catastrophic failure of materials. One way to mitigate this effect is to use functionally graded materials. Material properties vary continuously in a functionally graded material (FGM). However, they fluctuate discontinuously across adjoining layer in a laminated plate. Therefore, this advantage of a plate made of an FGM over a laminated plate eliminates the delamination mode of failure. FGMs are microscopically heterogeneous and made from isotropic materials such as metals and ceramics.

The use of FGMs in advanced engineering structures exposed to high temperature, were first reported in 1993 in Japan [5]. The analysis of the FG plates/panels has received considerable attention of the researchers in recent past. Finot and Suresh [13] presented a closed form solution based on the classical Kirchhoffs theory of thin plates for the analysis of multilayered and FG plates, subjected to thermal loading. Praveen and Reddy [12] obtained the static and dynamic responses of functionally graded ceramic-metal plate employing transverse shear deformation in which the effect of imposed temperature field was discussed in detail. Ng et al. [12] dealt with the parametric resonance of FG rectangular plates under harmonic in-plane loading. Vel and Batra [2, 3] presented an analytical solution for the three dimensional analysis of simply supported FG rectangular plate subjected to thermal and mechanical loadings. Roque et al. [6] used asymmetric collocation method with multiquadrics basis functions and a higher-order shear deformation theory (HSDT) to find static deformations and natural frequencies of square FG plates of various aspect rations. Wu et al. [10] attained an explicit solution for the nonlinear static and dynamic responses of FG rectangular plates. Their formulation was based on first-order shear deformation theory and Von-Karman nonlinear kinematics. Ferreira and Batra [3] provided a global collocation method for natural frequencies of FG plates by a meshless method with first order shear deformation theory. Matsunaga [8] presented a two-dimensional higher order deformation theory for the evaluation of displacements and stresses in FG plates exposed to thermal and mechanical loadings.

Skew and trapezoidal plates have quite a good number of applications in modern structures. Skew plate structures can be found frequently in modern construction in the form of reinforced slabs or stiffened plates. Such structures are widely used as floors in bridges, ship hulls, buildings, etc. Several researchers have addressed the linear and nonlinear static and dynamic problems of skew and trapezoidal plates [6–9, 14]. For plates with cutout, Ko [7] used anisotropic plate theory to evaluate the stress concentration factor for a single layer or laminated composite plates with central circular cutouts. Prasad and Shuart [11] presented a closed form solution for the moment distributions around holes in symmetric laminates subjected to bending moments. Doaust and Hoa [1] solved the case of circular and triangular cutouts and investigated the influence of blunt curvature and material properties on the state of stress around a triangular cutout in an infinite composite plate. Chai [15] presented finite element and some experimental results on the free vibration of symmetric composite plates with central hole. Huang and Sakiyama [4] proposed an approximate method for analyzing free vibration of rectangular plates with different cutouts. Liu et al [7] investigated static and free vibration analysis of laminated composite plates using the conforming radial point interpolation method. They also analyzed circular and non-circular cutouts. Kumar et al. [6] analyzed thick skew laminate with elliptical cutout subjected to non-linear temperature distribution.

Available literatures reveal that free vibration analysis of functionally graded nonumiform straight-sided plates with circular and non-circular cutouts in thermal environment, has not been performed yet. Therefore, the necessity of the study on such FG plates seems to be more obvious than before. This study is divided into two parts. The first part includes free vibration without any thermal loadings. In second part, free vibration analysis under thermal loading will be demonstrated. In the latter part, different shapes of plates with various cutouts will be investigated and the results will be tabulated. The idea of this paper is to analyze FG plates in ANSYS package. Introducing material properties which vary continuously is not available in ANSYS. In order to fulfill this problem, the plate is divided into several layers across the thickness, the properties of each layer are introduced and then these layers are glued to each other, as the whole plate. This could be performed by writing a code in MATLAB.

2 MATERIAL PROPERTIES

Material properties of the plates are assumed to vary across thickness. In the case of thermal environment, to compute the results for FG plates, the properties are as follows [8]:

Based on the linear distribution law, a typical material property P of the FG plate is obtained as,

$$P(z) = P_m + (P_c - P_m)(v_f) \tag{1}$$

where $0 < z < h$ and $v_f = (z/h)^p$, v_f is the volume fraction and p is the power law index which is assumed that the index parameter is $p = 1$ unless otherwise specified. Subscripts m and c refer to the metal and ceramic constituents that denote the material properties of the bottom and top surface of the plate, respectively. The material properties used in the present study are as follows:

$$E_m = 70 \text{ GPa}, \quad v_m = 0.3, \quad \alpha_m = 23 \times 10^{-6} 1/\text{K}, \quad k_m = 233\text{w/m.k}$$

$$E_c = 380 \text{ GPa}, \quad v_c = 0.3, \quad \alpha_c = 7.4 \times 10^{-6} 1/\text{K}, \quad k_c = 65\text{w/m.k}$$

Where E, ν, α and k are the Young's modulus, Poisson's ratio, thermal expansion coefficient and thermal conductivity, respectively. The density for Aluminum and Alumina are assumed to be 2700 and 3800 Kg/m^3, respectively and the thickness of plates is $h = 0.05$ m.

3 MODELING

3.1 Geometry

Straight-sided plates of various geometries such as rectangular, skew and trapezoidal plates with circular and non-circular cutouts were applied for free vibration analyses. The geometries are shown in Fig. 3.

3.2 Elements

The analysis was carried out using finite element package ANSYS. SOLID 45 was considered as a 3D 8-node structural solid element (Fig.1) for modal analysis under mechanical loading. The element has three degrees of freedom per node: translation in the nodal x, y and z directions. For thermal analysis SOLID 70 element (Fig.2) was chosen which has 3-D thermal conduction capability and eight nodes with a single degree of freedom, temperature, at each node. The element is applicable for doing a 3-D, steady-state or transient thermal analysis. Moreover, it can be applied to study heat and mass transport phenomena as well as flow in a constant velocity field. 2000 nodes were used in this analysis.

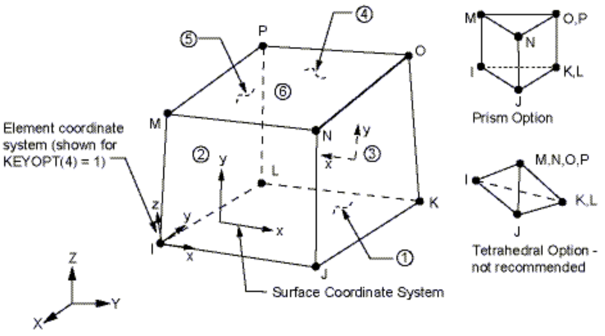


Figure 1 SOLID 186 geometry.

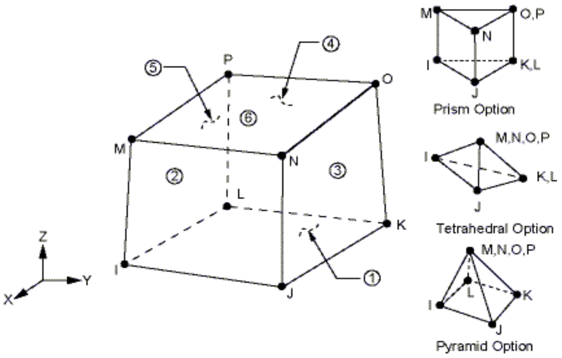


Figure 2 SOLID 70 geometry.

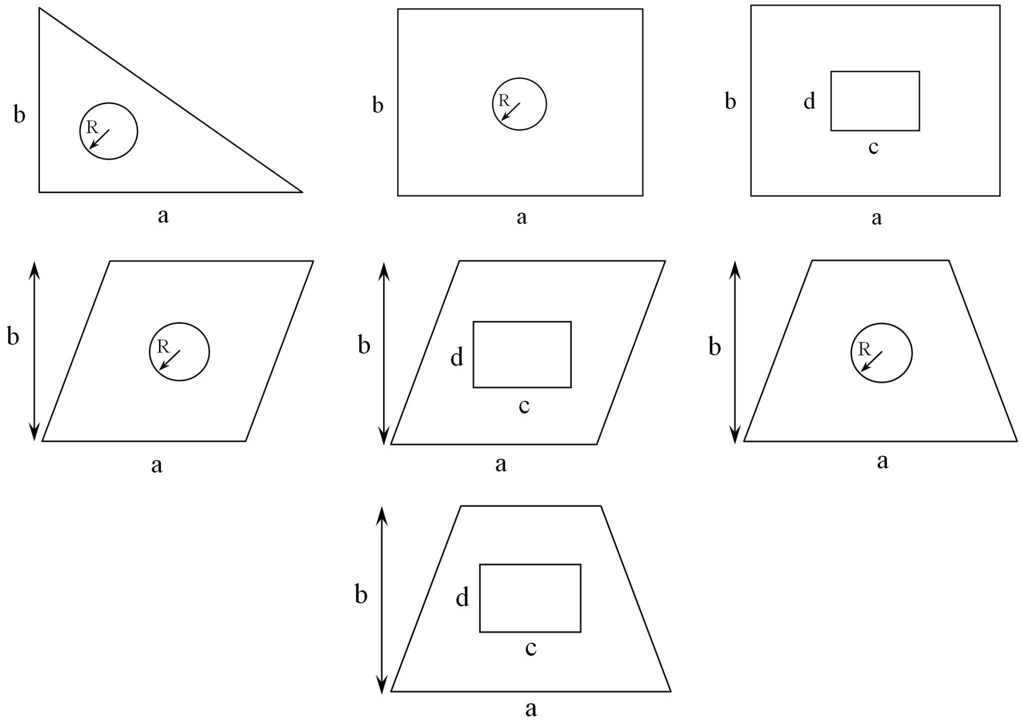


Figure 3 Plates with cutouts.

3.3 Method

It should be mentioned that subspace method was used in free vibration analysis. The subspace iteration method is described in detail by Bathe [1]. Wilson and Itoh's [15] suggestion about enhancement will be outlined subsequently.

3.4 Loading condition

Two different loading conditions were applied for free vibration analysis. It was assumed that the plate is stress free at temperature T_0. If the plate operates in a thermal environment, due to non-uniform temperature rise or mechanical constraints at its edges, some stresses will be produced in it. One of the usual thermal boundary conditions presumed in the literature is the prescribed temperature at the upper and lower surfaces of the beams and plates. Hence, these presumed boundary conditions for the present plate were considered.

$$T = T_m \quad \text{at} \quad z = 0 \quad \text{and} \tag{2a}$$

$$T = T_c \quad \text{at} \quad z = h \tag{2b}$$

It is assumed that the temperature at the lower and upper surface are T_m = 400 K and T_c = 1700 K, respectively, unless otherwise specified. T_0 was chosen to be 273 K. The other thermal boundary condition that was considered is the convection at the upper and lower surfaces. Film coefficients and bulk temperatures of the fluids at upper and lower surfaces are as follows,

$$T = T_m,\ h = h_m \quad \text{at} \quad z = 0 \quad \text{and} \tag{3a}$$

$$T = T_c,\ h = h_c \quad \text{at} \quad z = h \tag{3b}$$

It is assumed that the temperature and film coefficient at the lower and upper surface are T_m = 375 K, h_m = 15 and T_c = 1700 K, h_c = 20, respectively, unless otherwise specified. The boundary conditions are applied on external boundaries unless otherwise specified.

4 NUMERICAL RESULTS

In order to demonstrate the accuracy of methodology for free vibration analyses of irregular straight-sided plates, several plate samples (such as triangular, skew and trapezoidal plates) with cutouts are studied under different boundaries and loading conditions (Fig. 3).

In the following, results of free vibration analysis of isotropic and FGM plates with circular and non-circular cutout are demonstrated. For FGM plates, the effect of thermal environment is also investigated.

4.1 Isotropic plates

Free vibration of a simply supported square plate with a square cutout at the center, shown in fig. 3, is analyzed. The geometry and material parameters are length, a = 10; size ratio, c/a =

0.5; thickness ratio, $h/a = 0.01$; density, $\rho = 8000\ \mathrm{kg/m^2}$ and Young modulus, $E = 200$ GPa. The results were compared with those given by Huang and Sakiyama [4] and Liu et al [7] in Table 1. An appropriate compatibility is observed among the results.

A square plate with a circular cutout at the center is shown in Fig. 3, the length of this plate is a=10, the ratio of the radius to length is $r/a = 0.1$, and the thickness ratio is $h/a = 0.01$. The material properties are the same as above. Table 2 provides a comparison between present results and solution given by Huang and Sakiyama [4] and Liu et al [7] for a fully clamped plate.

Table 1 Nondimensional frequencies of isotropic square plate with square cutout at the center (simply support for external boundaries, $\varpi = [\rho h\omega^2 a^4/(D(1-\nu^2))]$,$h/a = 0.01$).

Mode	**Present**	**Liu et al.** [7]	**Huang et al.** [4]
1	4.92	4.97	4.84
2	6.43	6.48	6.43
3	6.43	6.48	6.44
4	8.59	8.55	8.49
5	8.69	8.87	8.87
6	10.73	10.72	10.81
7	10.73	10.77	10.83
8	12.23	12.04	12.29
9	13.33	13.37	13.53
10	14.40	14.18	14.11

Table 2 Nondimensional frequencies of isotropic square plate with circular cut out at the center (simply support for external boundaries, $\varpi = [\rho h\omega^2 a^4/(D(1-\nu^2))]$, $h/a = 0.01$, $r/a = 0.1$).

Mode	**Present**	**Liu et al.** [7]	**Huang et al.** [4]
1	6.17	6.15	6.24
2	8.62	8.58	8.46
3	8.62	8.63	8.46
4	10.48	10.42	10.23
5	11.52	11.41	11.72
6	12.01	11.84	12.30
7	12.96	12.83	13.04
8	12.96	12.84	13.04

4.2 Functionally graded plates

Free vibrations of FG plates were analyzed for different boundary conditions.

4.2.1 Free vibration results without thermal environment

In the following examples, free vibrations of various plates with different types of cutout are analyzed. Functionally graded skew plate with circular cutout at location ($x/a = 0.45$, $y/a = 0.75$) with length a=1 m, radius ratio $r/a = 0.25$ is presented. Skew plate with skew angle $\theta = 30°$ and length ratio $a/b = 1$ is used (Table 3).

Table 3 Natural frequencies of FG skew plate with circular cutout (fully clamped for internal and external boundaries, $\theta = 30°$, no thermal effect).

Mode	**Frequency**	**Mode**	**Frequency**
1	145.50	**6**	182.63
2	145.51	**7**	182.64
3	145.53	**8**	182.65
4	145.55	**9**	182.68
5	145.67	**10**	182.83

Free vibration analysis of functionally graded trapezoidal plates with a rectangular cutout at location (x/a =0.25, y/a =0.25) with length a=1 m, size ratios $c/a = 0.4$ and $d/a = 0.15$ is investigated (Table 4).

Table 4 Natural frequencies of FG skew plate with circular cutout (fully clamped for internal and external boundaries, $\theta = 30°$, no thermal effect).

Mode	**Frequency**	**Mode**	**Frequency**
1	987.0	**6**	2794.8
2	1180.6	**7**	2857.3
3	1751.3	**8**	3448.4
4	2281.5	**9**	3535.4
5	2577.4	**10**	3617.6

In Figure 5, an FG trapezoidal plate with a circular cutout at location ($x/a = 0.5$, $y/a = 0.4$) with geometry parameters as length a=1 m, length ratio $b/a = 0.45$, thickness ratio $h/a = 0.7$ and radius ratio $r/a = 0.15$ is analyzed.

4.2.2 Thermal effect on free vibration

In this section, temperature gradient effect on free vibration of FG non-uniform straight-sided plates with cutout is analyzed. Frequencies are calculated for first ten modes. Thermal boundary conditions are prescribed in section 3.4.

Square plates with cutout In Table 5, free vibration analyses of FG square plate with circular cutout at the center of plate subjected to constant temperature is investigated. Same geometry is studied for a plate subjected to convectional loading (Table 6).

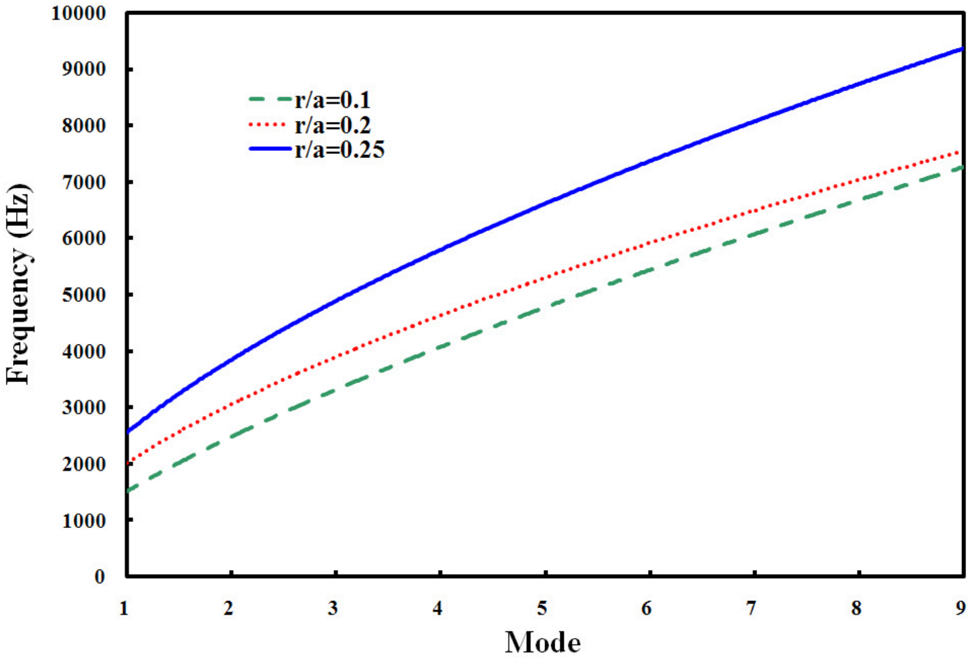


Figure 4 Natural frequencies of FG trapezoidal plate with different circular cutouts (fully clamped boundary condition, no thermal effect).

Table 5 Natural frequencies of FG square plate with circular cutout subjected to constant temperature (fully clamped boundary condition, $a/b = 1$, $r/a = 0.1$, $T_c = 1700$ K).

Mode	Frequency	Mode	Frequency
1	581.24	**6**	2481.3
2	920.03	**7**	2485.7
3	920.69	**8**	2984.2
4	1414.7	**9**	3697.4
5	1815.8	**10**	3704.7

In Table 7, natural frequencies of an FG square plate with square cutout at the center of the plate subjected to constant temperature are discussed. Same geometry is analyzed for a plate subjected to convectional loading (Table 8).

Skew plates with cutout FG skew plate with circular cutout at location ($x/a = 0.45, y/a = 0.75$) subjected to constant temperature are presented in Figure 5. This plate is also studied under convectional loading. The effect of bulk temperatures and film coefficients of the fluids are discussed in Figure 5 and Table 9, respectively.

Table 6 Natural frequencies of FG square plate with circular cutout subjected to convectional loading (fully clamped boundary condition, $a/b = 1$, $r/a = 0.1$, $T_c = 1900$ K).

Mode	Frequency	Mode	Frequency
1	233.4	**6**	2108.3
2	580.5	**7**	2113.4
3	581.6	**8**	2698.2
4	1005.0	**9**	3415.2
5	1461.5	**10**	3449.8

Table 7 Natural frequencies of an FG square plate with square cutout subjected to constant temperature (simply supported boundary condition, $a/b = 1$,$c/a = 0.5$, $T_c = 1700$ K).

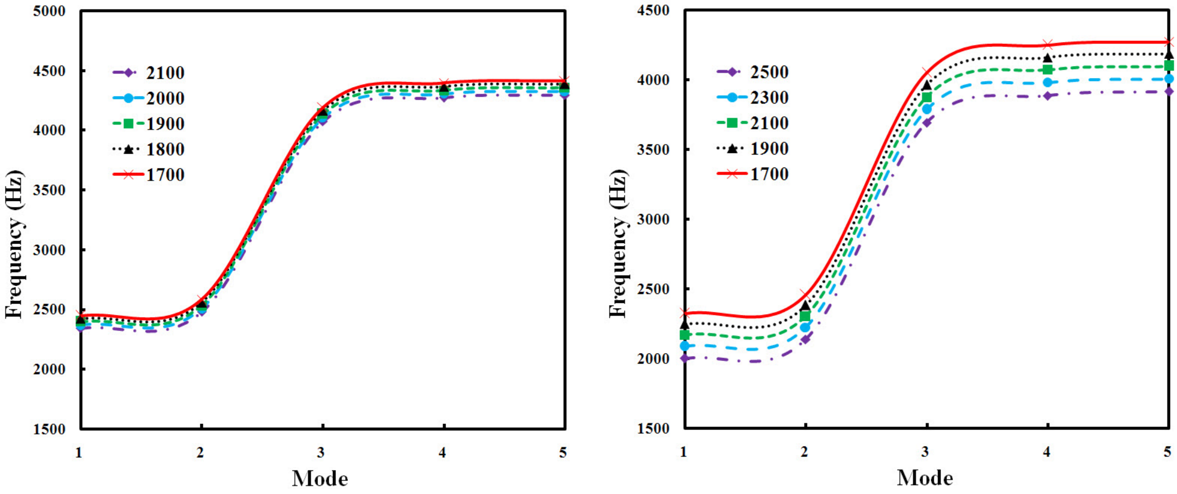
Mode	Frequency	Mode	Frequency
1	1196.7	**6**	2414.7
2	1348.0	**7**	2416.4
3	1349.1	**8**	3105.4
4	1764.1	**9**	3411.6
5	1777.4	**10**	4123.3

Table 8 Natural frequencies of an FG square plate with square cutout subjected to convectional loading (simply supported boundary condition, $a/b = 1$,$c/a = 0.5$, $T_c = 1900$ K).

Mode	Frequency	Mode	Frequency
1	1113.9	**6**	2182.4
2	1233.0	**7**	2184.5
3	1234.2	**8**	2830.8
4	1540.9	**9**	3155.2
5	1645.9	**10**	3904.9

Table 9 Parameter study for natural frequencies of FG skew plate with circular cutout subjected to convectional loading vs. film coefficients (fully clamped external and internal boundary condition, $a/b = 1$, $r/a = 0.25$, $\theta = 30°$).

Mode	h_c			
	10	**20**	**30**	**40**
1	1967.7	2363.5	2166.5	2047.6
2	2106.1	2498.8	2302.6	2184.9
3	3658.1	4096.7	3873.9	3743.7
4	3852.2	4296.6	4070.5	3938.7
5	3880.8	4318.7	4095.9	3966.0



(a) (b)

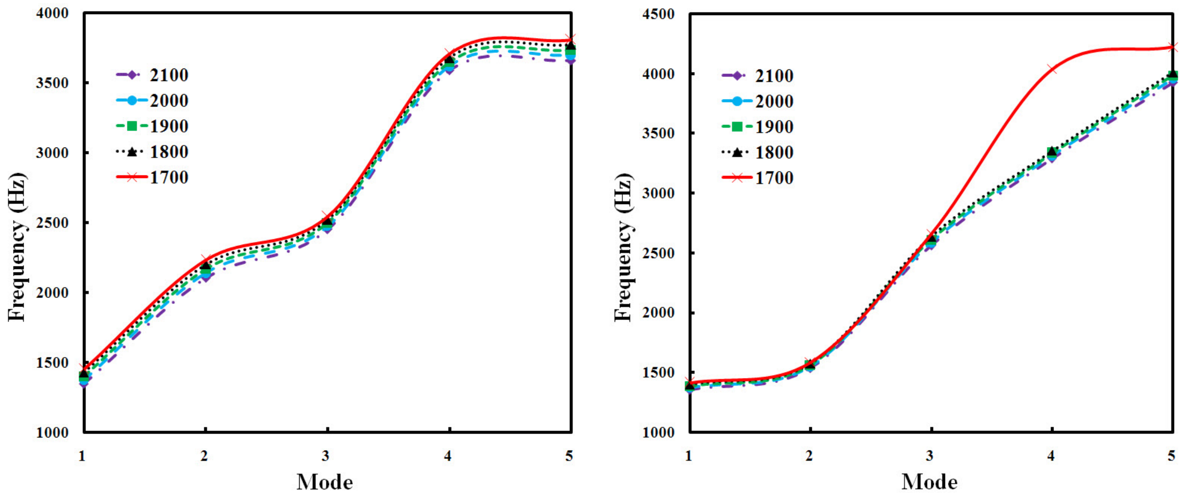
Figure 5 Parameter study for natural frequencies of FG skew plate with circular cutout vs. T_c (fully clamped external and internal boundary condition, $a/b = 1$, $r/a = 0.25$, $\theta = 30°$) **a)** subjected to constant temperature **b)** subjected to convectional loading.

Trapezoidal plate with cutout Natural frequencies of FG trapezoidal plate with circular cutout at location ($x/a = 0.5$, $y/a = 0.4$) subjected to constant temperature are calculated and presented in Table 10. Parameter study of this plate is investigated in Figure 6. It can be seen that increasing temperature at upper surface will decrease the natural frequencies of the plate. Same plate at location ($x/a = 0.25$, $y/a = 0.25$) with rectangular cutout subjected to constant temperature is presented in Table 11. Parameter study of this plate with rectangular cutout is tabulated in Figure 6. Again frequency decrease can be seen by increase in temperature at upper surface.

Table 10 Natural frequencies of FG trapezoidal plate with circular cutout subjected to constant temperature (C-S-C-S boundary condition, $b/a = 0.4$, $h/a = 0.75$, $r/a = 0.15$, $T_c = 1900$ K).

Mode	Frequency	Mode	Frequency
1	1399.2	**6**	4364.8
2	2167.0	**7**	5047.2
3	2496.2	**8**	5767.1
4	3645.5	**9**	5985.6
5	3733.6	**10**	6036.5

Trapezoidal plate with circular cutout is subjected to convectional loading at its upper and lower surfaces. Natural frequencies are calculated and presented in Table 12. It is shown that increasing bulk temperature on upper surface will decrease natural frequencies. In Table 13 and 14, the effect of thermal environment on the frequencies of trapezoidal plates with rectangular cutouts is investigated.



(a) (b)

Figure 6 Parameter study for natural frequencies of FG trapezoidal plate with circular cutout subjected to constant temperature vs. temperature ($b/a = 0.4$, $h/a = 0.75$) **a)** circular cutout (C-S-C-S boundary condition, $r/a = 0.15$) **b)** rectangular cutout (fully clamped boundary condition, $c/a = 0.5$, $d/a = 0.15$).

Table 11 Natural frequencies of FG trapezoidal plate with rectangular cutout subjected to constant temperature (fully clamped boundary condition, $b/a = 0.4$, $h/a = 0.75$, $c/a = 0.5$, $d/a = 0.15$, $T_c = 2000$ K).

Mode	**Frequency**	**Mode**	**Frequency**
1	1368.3	**6**	4183.0
2	1548.1	**7**	5605.1
3	2584.2	**8**	5954.4
4	3309.5	**9**	6290.8
5	3952.6	**10**	6521.9

Table 12 Natural frequencies of FG trapezoidal plate with circular cutout subjected to convectional loading vs. bulk temperatures (fully clamped boundary condition, $b/a = 0.4$, $h/a = 0.75$, $r/a = 0.15$).

Mode	T_c [K]				
	1700	**1900**	**2100**	**2300**	**2500**
1	1332.3	1287.7	1241.2	1192.7	1141.8
2	1519.2	1484.7	1449.4	1413.2	1375.9
3	2525.7	2453.8	2379.6	2302.8	2223.1
4	3255.1	3190.2	3123.9	3055.9	2986.2
5	3883.4	3798.1	3710.0	3619.1	3525.2

Table 13 Natural frequencies of FG trapezoidal plate with rectangular cutout subjected to convectional loading (fully clamped boundary condition, $b/a = 0.4$, $h/a = 0.75$, $c/a = 0.5$, $d/a = 0.15$, $T_c = 2300$ K).

Mode	**Frequency**	**Mode**	**Frequency**
1	1192.7	**6**	4034.8
2	1413.2	**7**	5253.1
3	3202.8	**8**	5697.2
4	3055.9	**9**	6041.9
5	3619.1	**10**	6390.2

Table 14 Parameter study for natural frequencies of FG trapezoidal plate with rectangular cutout subjected to convectional loading (fully clamped boundary condition, $b/a = 0.4$, $h/a = 0.75$, $c/a = 0.5$, $d/a = 0.15$).

Mode	T_c [K]				
	1700	**1900**	**2100**	**2300**	**2500**
1	1332.3	1287.7	1241.2	1192.7	1141.8
2	1519.2	1484.7	1449.4	1413.2	1375.9
3	2525.7	2453.8	2379.6	2302.8	2223.1
4	3255.1	3190.2	3123.9	3055.9	2986.2
5	3883.4	3798.1	3710.0	3619.1	3525.2

5 CONCLUSION

This analysis has been presented for free vibration of functionally graded non-uniform straight-sided plates with circular and non-circular cutouts in thermal environment. The accuracy of the method is demonstrated by comparing the results with those of the existing solutions. Moreover, parameter studies have been performed to show the effects of cutout size, length-to-thickness ratio, type of loading and boundary condition for the plate on natural frequencies. Some of the results of present work are as follows:

1. For prescribed temperatures on upper and lower surfaces, increasing in temperature on upper surface will increase frequencies of FG square/skew/trapezoidal plates with cutout.
2. For convectional loading on upper and lower surfaces, increasing in bulk temperature on upper surface will increase frequencies for FG square/skew/trapezoidal plates with cutout.
3. Increasing film coefficients of fluid on upper surface for FG skew plate with circular cutout will decrease natural frequencies.

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