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Propagation of SH Waves in a Double Non-Homogeneous Crustal Layers of Finite Depth Lying Over an Homogeneous Half-Space

Abstract

The present paper studies the Propagation of SH waves in a double non-homogeneous crustal layers lying over an isotropic homogeneous half-space, where upper layer ((i.e. rigidity and density varying trigonometrically with depth) and intermediate layer (i.e. rigidity and density varying parabolically with depth). The wave velocity equation has been obtained. Closed form solutions have been derived separately for the displacements in two non-homogeneous crustal layers and lower half-space. The dispersion curves are depicted by means of graphs for different values of non-homogeneity parameters and thickness ratio for layers.

Keywords

SH waves, Whittaker functions, Non homogeneity, Differential equations.

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1 INTRODUCTION

The formulations and solutions of many problems of linear wave-propagation for homogeneous media are available in the literature of continuum mechanics of solids. In recent years, however, sufficient interest has arisen in the problem connected with bodies whose mechanical properties are functions of space, i.e. non-homogeneous bodies. This interest is mainly due to the advent of solid rocket propellants, polymeric materials and growing demand of engineering and industrial applications.

The propagation of surface waves in elastic media is of considerable importance in earth-quake engineering and seismology on account of occurrence of stratification in the earth crust, as the earth is made up of different layers. As a result, the theory of surface waves has been developed by Stoneley [1924], Bullen [1965], Ewing et al. [1957], Hunters [1970] and Jeffreys [1970].

Many results of theoretical and experimental studies revealed that a real earth is considerably more complicated than the models presented earlier. This has led to a need for more realistic representation as a medium through which seismic waves propagate. The wave propagation in crystalline media plays a very interesting role in geophysics and also in ultrasonic and signal processing. Monoclinic medium is an example of such medium, keeping in the mind the fact that the non-homogeneity characteristic is one of the most generalized elastic conditions inside the earth, many authors have studied the propagation of different waves in different media with non-homogeneity.

Sezawa [1927] studied the dispersion of elastic waves propagated on curved surfaces. The transmission of elastic waves through a stratified solid medium was studied by Thomson [1950]. Haskell [1953] studied the dispersion of surface waves in multilayered media. Biot [1965] studied the influence of gravity on Rayleigh waves, assuming the force of gravity to create a type of initial stress of hydrostatic nature and the medium to be incompressible.

Propagation of Love waves in a non-homogeneous stratum of finite depth sandwiched between two semi infinite isotropic media has been studied earlier by Sinha [1967]. Wave propagation in a thin two-layered laminated medium with couple under initial stress was studied by Roy [1984]. Datta [1986] studied the effect of gravity on Rayleigh wave propagation in a homogeneous, isotropic elastic solid medium. Effects of irregularities on the propagation of guided SH waves has been studied by Chattopadhyay et al [1983]. Goda [1992] studied the effect of non-homogeneity and anisotropy on Stoneley waves. Gupta et al. [2012] investigated the influence of linearly varying density and rigidity on torsional surface waves in inhomogeneous crustal layer.

Some of the recent notable works on propagation of seismic waves in various media with different geometries were done by Chattopadhyay et al [2009, 2010, 2010].

Recently Sethi et al. [2013] investigated the surface waves in homogeneous visco-elastic media of higher order under the influence of surface stresses.

In the present problem, we have considered the propagation of SH waves in a double non-homogeneous crustal layers lying over an isotropic homogeneous half-space. The expression for displacement and dispersion equation is found in the closed form. The dispersion curves are depicted by means of graphs for different values of non-homogeneity parameters and thickness ratio. The influence of non-homogeneity parameters, wave number and thickness ratio of the layers on the dimensionless phase velocity has been studied.

Here, the non-homogeneity in the upper layer can be taken as $\mu = \mu_1(1 - \sin\alpha z)$; $\rho = \rho_1(1 - \sin\alpha z), \alpha > 0.$ Also, the non-homogeneity in intermediate layer can be taken as, $\mu = \mu_2(1 + nz)^2$, $\rho = \rho_2(1 + nz)^2, n > 0.$ and the lower half-space is homogeneous, where α, n are all constants and having the dimensions that are inverse of length.

2 FORMULATION OF THE PROBLEM

Let the density and rigidity of layers (medium-I) and (medium-II) are ρ_1, μ_1 and ρ_2, μ_2 respectively and ρ_3, μ_3 are the density and rigidity of homogeneous half-space (medium-III).

The thicknesses of the first and second layers are H_1 and H_2 respectively.

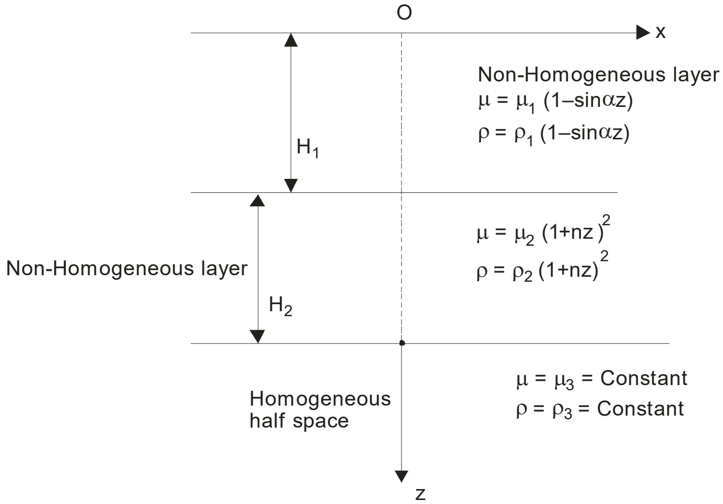

Figure 1: Geometry of the problem.

Let us consider x-axis along the direction of the wave propagation and z-axis is taken vertically downwards. Let u, v, w are displacements along x, y, z axis respectively.

Let (u_1, v_1, w_1), (u_2, v_2, w_2), (u_3, v_3, w_3) are the displacements for the layers (medium-I) and (medium-II) and half-space (medium-III) respectively.

For SH wave propagation we have,

$$u = w = 0 \text{ and } v = v\,(x, z, t\,) \tag{1}$$

The equation governing the propagation of Love wave in homogeneous isotropic elastic medium in the absence of body forces are

$$\begin{aligned}
&\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx} = \rho\frac{\partial^2 u}{\partial t^2}, \\
&\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy} = \rho\frac{\partial^2 v}{\partial t^2}, \\
&\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz} = \rho\frac{\partial^2 w}{\partial t^2},
\end{aligned} \tag{2}$$

Also, Hooke's law for isotropic medium,

$$\tau_{ij} = \lambda\Delta\delta_{ij} + 2\mu\varepsilon_{ij,} \tag{3}$$

where λ, μ are lame's constants and Δ be the dilatation.

$$\text{Also}, \varepsilon_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right]. \tag{4}$$

3 SOLUTION FOR UPPER LAYER (MEDIUM-I)

In view of equations (1), (3) and (4), equation of motion (2) gives the non-vanishing equations of motion for propagation of SH wave in upper layer (medium-II) as

$$\frac{\partial}{\partial x}\left(\mu \frac{\partial u_1}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu \frac{\partial u_1}{\partial z}\right) = \rho \frac{\partial^2 u_1}{\partial t} \tag{5}$$

For a wave propagating in x-direction,

$$u_1 = W(z)\, e^{i(kx - wt)} \tag{6}$$

equation (5) reduces to,

$$\frac{d^2 W}{dz^2} + \frac{1}{\mu}\frac{d\mu}{dz}\frac{dW}{dz} + K^2\left(\frac{\rho c^2}{\mu} - 1\right)W = 0, \tag{7}$$

where $c = \dfrac{\omega}{K}$ be the velocity of SH waves.

Introducing $W = \dfrac{W_1}{\sqrt{\mu}}$ in equation (7), to eliminate $\dfrac{dW}{dz}$, we get

$$\frac{d^2 W_1}{dz^2} - \frac{1}{2\mu}\frac{d^2\mu}{dz^2} W_1 + \frac{1}{4\mu^2}\left(\frac{d\mu}{dz}\right)^2 W_1 + K^2\left(\frac{\rho c^2}{\mu} - 1\right) W_1 = 0 \tag{8}$$

We take variations in rigidity and density as

$$\mu = \mu_1(1 - \sin \alpha z);\ \rho = \rho_1(1 - \sin \alpha z), \tag{9}$$

Here $\alpha > 0$ for non-homogeneity and $\alpha = 0$ for homogeneity

Introducing (9) in equation (8), we obtain

$$\frac{d^2 W_1}{dz^2} + \xi^2 W_1 = 0;$$
$$\text{where } \xi^2 = K^2\left(\frac{\alpha^2}{4K^2} + \frac{c^2}{\beta_1^{\,2}} - 1\right), \beta_1 = \sqrt{\frac{\mu_2}{\rho_2}} \tag{10}$$

Thus solution for equation (10) can be taken as

$$W_1 = A \cos \xi z + B \sin \xi z$$

where A and B are arbitrary constants.

Thus displacement component for non-homogenous layer (medium-II) is given by

$$u_1(x,z,t) = \frac{A\cos\xi z + B\sin\xi z}{(1-\sin\alpha z)^{1/2}} e^{i(Kx-wt)} \tag{11}$$

4 SOLUTION FOR INTERMEDIATE LAYER (MEDIUM-II)

In view of equations (1), (3) and (4), equation of motion (2) gives the non-vanishing equations of motion for propagation of SH wave in upper layer (medium-II) as

$$\frac{\partial}{\partial x}\left(\mu\frac{\partial u_2}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial u_2}{\partial z}\right) = \rho\frac{\partial^2 u_2}{\partial t^2} \tag{12}$$

For a wave propagating in x-direction,

$$u_2 = V(z)\, e^{i(kx-wt)} \tag{13}$$

equation (13) reduces to,

$$\frac{d^2V}{dz^2} + \frac{1}{\mu}\frac{d\mu}{dz}\frac{dV}{dz} + K^2\left(\frac{\rho c^2}{\mu} - 1\right)V = 0 \tag{14}$$

Introducing $V = \frac{V_1}{\sqrt{\mu}}$ in equation (14), to eliminate $\frac{dV}{dz}$, we get

$$\frac{d^2V_1}{dz^2} - \frac{1}{2\mu}\frac{d^2\mu}{dz^2}V_1 + \frac{1}{4\mu^2}\left(\frac{d\mu}{dz}\right)^2 V_1 + K^2\left(\frac{\rho c^2}{\mu} - 1\right)V_1 = 0 \tag{15}$$

We take variations in rigidity and density as

$$\mu = \mu_2(1+nz)^2, \rho = \rho_2(1+nz)^2 \tag{16}$$

Here $n > 0$ for non-homogeneity and $n = 0$ for homogeneity

Introducing (16) in equation (15), we get

$$\frac{d^2V_1}{dz^2} + m^2V_1 = 0; \quad \text{where } m^2 = K^2\left(\frac{c^2}{\beta_2{}^2} - 1\right), \beta_2 = \sqrt{\frac{\mu_2}{\rho_2}} \qquad 17)$$

Thus solution for equation (17) can be taken as

$$V_1 = C\cos mz + D\sin mz,$$

where C and D are arbitrary constants.

Thus displacement component for non-homogeneous intermediate layer (medium-II) is given by

$$u_2(x,z,t) = \frac{C\cos mz + D\sin mz}{(1+nz)} e^{i(Kx-wt)} \tag{18}$$

5 SOLUTION FOR SEMI INFINITE HALF SPACE (MEDIUM-III)

In view of equations (1), (3) and (4), equation of motion (2) gives the non-vanishing equations of motion for propagation of SH wave in upper layer (medium-II) as

$$\frac{\partial}{\partial x}\left(\mu\frac{\partial u_3}{\partial x}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial u_3}{\partial z}\right)=\rho\frac{\partial^2 u_3}{\partial t^2} \tag{19}$$

For a wave propagating in x-direction,

$$u_3 = U(z)\, e^{i(kx-wt)} \tag{20}$$

equation (20) reduces to,

$$\frac{d^2U}{dz^2}+\frac{1}{\mu}\frac{d\mu}{dz}\frac{dU}{dz}+K^2\left(\frac{\rho c^2}{\mu}-1\right)U=0 \tag{21}$$

Introducing $U = \dfrac{U_1}{\sqrt{\mu}}$ in equation (21), to eliminate $\dfrac{dU}{dz}$, we get

$$\frac{d^2U_1}{dz^2}-\frac{1}{2\mu}\frac{d^2\mu}{dz^2}U_1+\frac{1}{4\mu^2}\left(\frac{d\mu}{dz}\right)^2U_1+K^2\left(\frac{\rho c^2}{\mu}-1\right)U_1=0 \tag{22}$$

We take variations in rigidity and density as

$$\mu=\mu_3,\ \rho=\rho_3. \tag{23}$$

Introducing (23) in equation (22), we get

$$\begin{gathered}\frac{d^2U_1}{dz^2}+p^2U_1=0;\\ \text{where } p^2=K^2\ (1-\frac{c^2}{{\beta_3}^2}),\beta_3=\sqrt{\frac{\mu_3}{\rho_3}}\end{gathered} \tag{24}$$

Thus solution for equation (24) can be taken as

$$U_1 = Ee^{-pz}+F\,e^{pz},$$

where E and F are arbitrary constants.

Thus displacement component for homogenous half-space (medium-III) is given by

$$u_3(x,z,t)=Ee^{-pz}e^{i(Kx-wt)}. \tag{25}$$

6 BOUNDARY CONDITIONS

The boundary conditions are as follows:

$$
\begin{aligned}
&\text{(i) } \mu_1 \frac{\partial u_1}{\partial z} = 0 \text{ at } z = 0, \\
&\text{(ii) } \mu_1 \frac{\partial u_1}{\partial z} = \mu_2 \frac{\partial u_2}{\partial z} \text{ at } z = H_1, \\
&\text{(iii) } u_1 = u_2 \text{ at } z = H_1, \\
&\text{(iv) } \mu_2 \frac{\partial u_2}{\partial z} = \mu_3 \frac{\partial u_3}{\partial z} \text{ at } z = H_1 + H_2 \\
&\text{(v) } u_2 = u_3 \text{ at } z = H_1 + H_2
\end{aligned}
$$

Using all the boundary conditions (i) to (v) and after simplification, we have

$$\frac{\alpha}{2} A + B\xi = 0. \tag{26}$$

$$
\begin{aligned}
&\mu_1 \sqrt{1 - \sin \alpha H_1} \left[A \left(\mathrm{Cos}\xi H_1 \frac{\alpha}{2} \frac{\cos \alpha H_1}{1 - \mathrm{Sin}\alpha H_1} - \xi \mathrm{Sin}\xi H_1 \right) + B \left(\frac{\alpha}{2} \frac{\cos \alpha H_1}{1 - \sin \alpha H_1} \sin \xi H_1 + \xi \cos \xi H_1 \right) \right] \\
&= \mu_2 (1 + nH_1) \left[C \left(\frac{-n}{1 + nH_1} \cos mH_1 - m \sin mH_1 \right) + D \left(\frac{-n}{1 + nH_1} \sin mH_1 + m \cos mH_1 \right) \right]
\end{aligned} \tag{27}
$$

$$\frac{A \cos \xi H_1 + B \sin \xi H_1}{\sqrt{1 - \sin \alpha H_1}} = \frac{C\, \mathrm{Cos} mH_1 + D \sin mH_1}{(1 + nH_1)}. \tag{28}$$

$$\mu_2 (1 + n(H_1 + H_2)) \left[C \left\{ \frac{-n}{1 + n(H_1 + H_2)} \cos m(H_1 + H_2) - m \sin m(H_1 + H_2) \right\} + D \left\{ \frac{-n}{1 + n(H_1 + H_2)} \sin m(H_1 + H_2) + m \cos m(H_1 + H_2) \right\} \right] = -\mu_3 p e^{-p(H_1 + H_2)} E \tag{29}$$

$$\frac{C \cos m(H_1 + H_2) + D \sin m(H_1 + H_2)}{1 + n(H_1 + H_2)} = E e^{-p(H_1 + H_2)} \tag{30}$$

Eliminating A, B, C, D and E from (26), (27), (28), (29) and (30), we get

$$\mathrm{Det}\,(G_{ij}) = 0, \text{ where } i, j = 1, 2, 3, 4, 5. \tag{31}$$

where

$$
\begin{aligned}
&G_{11} = \frac{\alpha}{2}, G_{12} = \xi,\ G_{13} = G_{14} = G_{15} = 0, \\
&G_{21} = \mu_1 \sqrt{1 - \mathrm{Sin}\alpha H_1} \left[\frac{\alpha}{2} \frac{\cos \alpha H_1}{1 - \sin \alpha H_1} \cos \xi H_1 - \xi \sin \xi H_1 \right], \\
&G_{22} = \mu_1 \sqrt{1 - \mathrm{Sin}\alpha H_1} \left[\frac{\alpha}{2} \frac{\cos \alpha H_1}{1 - \sin \alpha H_1} \sin \xi H_1 - \xi \cos \xi H_1 \right], \\
&G_{23} = -\mu_2 (1 + nH_1) \left[\frac{-n}{1 + nH_1} \cos mH_1 - m \sin mH_1 \right], \\
&G_{24} = -\mu_2 (1 + nH_1) \left[\frac{-n}{1 + nH_1} \sin mH_1 + m \cos mH_1 \right], G_{25} = 0,
\end{aligned}
$$

$$G_{31}=\frac{\operatorname{Cos}\xi H_1}{\sqrt{1-\operatorname{Sin}\alpha H_1}}, G_{32}=\frac{\operatorname{Sin}\xi H_1}{\sqrt{1-\operatorname{Sin}\alpha H_1}}, G_{33}=\frac{-\operatorname{Cos}mH_1}{1+nH_1},$$

$$G_{34}=\frac{-\sin mH_1}{1+nH_1}, G_{35}=0,$$

$$G_{41}=G_{42}=0, G_{43}=\mu_2(1+n(H_1+H_2))\left[\frac{-n}{1+n(H_1+H_2)}\operatorname{Cos}m(H_1+H_2)-m\sin m(H_1+H_2)\right],$$

$$G_{44}=\mu_2(1+n(H_1+H_2))\left[\frac{-n}{1+n(H_1+H_2)}\sin m(H_1+H_2)+m\cos m(H_1+H_2)\right],$$

$$G_{45}=\mu_3 p e^{-p(H_1+H_2)},$$

$$G_{51}=0=G_{52}, G_{53}=\frac{\operatorname{Cos}m(H_1+H_2)}{1+n(H_1+H_2)}, G_{54}=\frac{\operatorname{Sin}m(H_1+H_2)}{1+n(H_1+H_2)},$$

$$G_{55}=\mu_3 e^{-p(H_1+H_2)}.$$

After simplification equation (31), we have

$$\tan\xi H_1=\frac{[1+T_2 S]\xi}{\frac{\alpha}{2}+T_1 S}, \tag{32}$$

where

$$S=\frac{T_7-T_6\tan mH_1}{(T_4T_6+T_7T_3)T_5},$$

$$T_1=\xi^2+\frac{\alpha^2}{4}\frac{\operatorname{Cos}\alpha H_1}{1-\operatorname{Sin}\alpha H_1},$$

$$T_2=\frac{\alpha}{2}\left(\frac{\operatorname{Cos}\alpha H_1}{1-\operatorname{Sin}\alpha H_1}-1\right),$$

$$T_3=\frac{n}{1+nH_1}+m\tan mH_1,$$

$$T_4=\frac{-n}{1+nH_1}\tan mH_1+m,$$

$$T_5=\frac{\mu_2}{\mu_1}\frac{(1+nH_1)^2}{1-\sin\alpha H_1},$$

$$T_6=-n\left(1+n(H_1+H_2)\right)+\frac{\mu_3}{\mu_2}p-m(1+n(H_1+H_2))^2\tan m(H_1+H_2),$$

$$T_7=\left[-n(1+n(H_1+H_2))+\frac{\mu_3}{\mu_2}p\right]\tan m(H_1+H_2)+m[1+n(H_1+H_2)]^2.$$

which gives the period equation for propagation of SH waves in a non-homogeneous isotropic two layers lying over homogeneous isotropic semi-infinite medium.

7 PARTICULAR CASES

Case (I): When upper layer is homogeneous i.e. $\alpha=0$, and intermediate layer is non-homogeneous, i.e. $n\neq0$, dispersion equation (32) takes the form,

$$\tan\left[KH_1\sqrt{\frac{c^2}{\beta_1^{\,2}}-1}\right]=\frac{(T_4T_6+T_7T_3)T_5}{K(T_7-T_6\tan mH_1)\sqrt{\frac{c^2}{\beta_1^{\,2}}-1}}, \tag{33}$$

where,

$$T_3=\frac{n}{1+nH_1}+m\tan mH_1,$$

$$T_4=\frac{-n}{1+nH_1}\tan mH_1+m,$$

$$T_5=\frac{\mu_2}{\mu_1}\frac{(1+nH_1)^2}{1-\sin\alpha H_1},$$

$$T_6=-n\big(1+n(H_1+H_2)\big)+\frac{\mu_3}{\mu_2}p-m(1+n(H_1+H_2))^2\tan m(H_1+H_2),$$

$$T_7=\left[-n(1+n(H_1+H_2))+\frac{\mu_3}{\mu_2}p\right]\tan m(H_1+H_2)+m\,[1+n(H_1+H_2)]^2.$$

which gives the wave velocity equation for propagation of SH waves in a homogeneous isotropic layer lying between two initially stressed non-homogeneous isotropic semi-infinite medium.

Case (II): When upper layer is non-homogeneous i.e. $\alpha\neq0$, and intermediate layer is homogeneous, i.e. $n=0$, dispersion equation (32) takes the form,

$$\tan\xi H_1=\frac{[1+T_2\,S]\xi}{\frac{\alpha}{2}+T_1S}, \tag{34}$$

where

$$S=\frac{T_7-T_6\tan mH_1}{(T_4T_6+T_7T_3)T_5},$$

$$T_1=\xi^2+\frac{\alpha^2}{4}\frac{\text{Cos}\alpha H_1}{1-\text{Sin}\alpha H_1},$$

$$T_2=\frac{\alpha}{2}\left(\frac{\text{Cos}\alpha H_1}{1-\text{Sin}\alpha H_1}-1\right),$$

$$T_3=m\tan mH_1,$$

$$T_4 = m,$$

$$T_5 = \frac{\mu_2}{\mu_1}\frac{1}{1-\sin\alpha H_1},$$

$$T_6 = \frac{\mu_3}{\mu_2}p - m\tan m(H_1+H_2),$$

$$T_7 = \frac{\mu_3}{\mu_2}p\tan m(H_1+H_2)+m.$$

which gives the dispersion relation for propagation of SH waves in a non-homogeneous isotropic layer lying between initially stressed Gibson half-space and initially stressed non-homogeneous isotropic semi-infinite medium.

Case (III): When both layers are homogeneous i.e. $\alpha=0$, $n=0$, dispersion equation (32) reduces to,

$$\tan[\xi H_1] = \frac{\mu_2}{\mu_1}\frac{m}{\xi}\left[\frac{\frac{\mu_3}{\mu_2}p - m\tan m(2H_1+H_2)}{\frac{\mu_3}{\mu_2}\tan mH_2 + m}\right]\left[\frac{1-\tan mH_1\tan m(H_1+H_2)}{1+\tan mH_1\tan m(H_1+H_2)}\right], \tag{35}$$

where,

$$\xi = K\sqrt{\frac{c^2}{\beta_1^{\,2}}-1},$$

$$m = K\sqrt{\frac{c^2}{\beta_2^{\,2}}-1},$$

$$p = K\sqrt{1-\frac{c^2}{\beta_3^{\,2}}}.$$

which gives the wave velocity equation for propagation of SH waves in a homogeneous isotropic layer lying over homogeneous isotropic semi-infinite medium.

Case (IV): In the absence of upper layer i.e. $H_1\to 0$, $n\neq 0$, and $H_2\neq 0$, dispersion equation (32) reduces to,

$$\tan\left[KH_2\sqrt{\frac{c^2}{\beta_2^{\,2}}-1}\right] = \frac{n^2(1+nH_2)KH_2\sqrt{\frac{c^2}{\beta_2^{\,2}}-1}}{n^2(1+nH_2)+(1+nH_2)^2K^2\left(\frac{c^2}{\beta_2^{\,2}}-1\right)-nK\frac{\mu_3}{\mu_2}\sqrt{1-\frac{c^2}{\beta_3^{\,2}}}}. \tag{36}$$

which gives the wave velocity equation for propagation of SH waves in a non-homogeneous isotropic layer lying over homogeneous isotropic semi-infinite medium.

Case (V): In the absence of upper layer i.e. $H_1\to 0$, $n=0$, and $H_2\neq 0$, dispersion equation (32) reduces to,

$$\tan\left[KH_2\sqrt{\frac{c^2}{\beta_2{}^2}-1}\right]=\frac{\mu_3\sqrt{1-\frac{c^2}{\beta_3{}^2}}}{\mu_2\sqrt{\frac{c^2}{\beta_2{}^2}-1}}. \tag{37}$$

which gives the wave velocity equation for propagation of SH waves in a homogeneous isotropic layer lying over homogeneous isotropic semi-infinite medium.

Case (VI): In the absence of intermediate layer i.e. $H_2 \to 0$, $\alpha \neq 0$, and $H_1 \neq 0$, dispersion equation (32) reduces to,

$$\tan\left(KH_1\sqrt{\frac{\alpha^2}{4}+\left(\frac{c^2}{\beta_1{}^2}-1\right)}\right)=\frac{\left[\frac{\alpha}{2}\left(\frac{\mathrm{Cos}\alpha H_1}{1-\mathrm{Sin}\alpha H_1}-1\right)+\frac{\mu_3}{\mu_1}K\sqrt{1-\frac{c^2}{\beta_3{}^2}}\right]K\sqrt{\frac{\alpha^2}{4}+\left(\frac{c^2}{\beta_3{}^2}-1\right)}}{K^2\left[\frac{\alpha^2}{4}+\left(\frac{c^2}{\beta_3{}^2}-1\right)\right]+\left(\frac{\alpha}{2}-\frac{\mu_3}{\mu_1}K\sqrt{1-\frac{c^2}{\beta_3{}^2}}\right)\frac{\alpha}{2}\frac{\mathrm{Cos}\alpha H_1}{1-\mathrm{Sin}\alpha H_1}}. \tag{38}$$

which gives the wave velocity equation for propagation of SH waves in a non-homogeneous isotropic upper layer lying over homogeneous isotropic semi-infinite medium.

Case(VII): In the absence of intermediate layer i.e. $H_2 \to 0$, $\alpha = 0$, and $H_1 \neq 0$, dispersion equation (32) reduces to,

$$\tan\left[KH_1\sqrt{\frac{c^2}{\beta_1{}^2}-1}\right]=\frac{\mu_3\sqrt{1-\frac{c^2}{\beta_3{}^2}}}{\mu_1\sqrt{\frac{c^2}{\beta_1{}^2}-1}}. \tag{39}$$

which gives the wave velocity equation for propagation of SH waves in a homogeneous upper isotropic layer lying over homogeneous isotropic semi-infinite medium.

8 NUMERICAL COMPUTATIONS AND DISCUSSION

To study the effect of various dispersion non-homogeneities and thickness ratio on the propagation of SH wave propagating in double non-homogeneous layers lying over an isotropic homogeneous semi-infinite media, where upper layer (i.e. rigidity and density varying trigonometrically with depth) and intermediate layer (i.e. rigidity and density varying parabolically with depth), phase velocity is calculated numerically for equation (32), we take the following data:

For non-homogeneous crustal layer (Medium-I) (Gubbins and Tierstein [1990])

$$\mu_1 = 7.84\times10^{10}\,\mathrm{N/m^2}, \rho_1 = 3535\,\mathrm{Kg/m^3}.$$

For non-homogeneous crustal layer (Medium-II) (Gubbins and Tierstein [1990])

$$\mu_2 = 6.34\times10^{10}\,\mathrm{N/m^2}, \rho_2 = 3364\,\mathrm{Kg/m^3}.$$

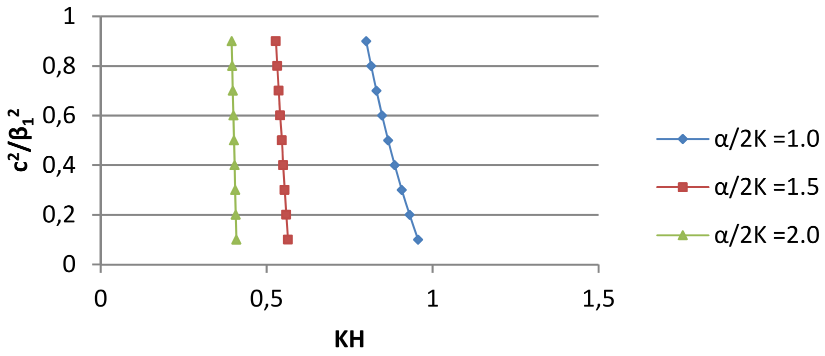

Figure 2: Variation of dimensionless phase velocity $(c/\beta_1)^2$ against dimensionless wave number KH demonstrating the influence of non-homogeneity associated with upper crustal layer for $n/K = 1.0$, $H_2/H_1 = 1.0$.

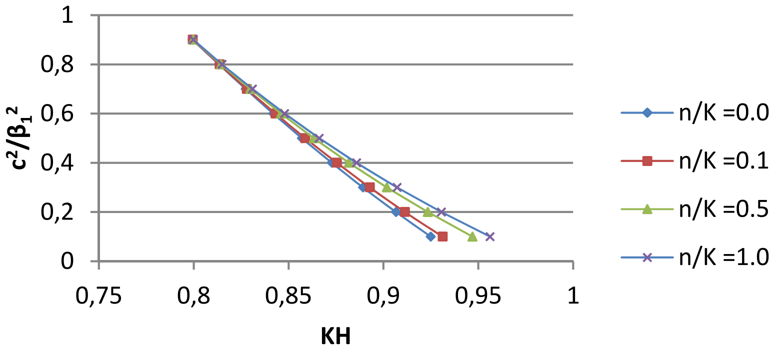

Figure 3: Variation of dimensionless phase velocity $(c/\beta_1)^2$ against dimensionless wave number KH demonstrating the influence of non-homogeneity associated with intermediate crustal layer for $\alpha/2K = 1.0$, $H_2/H_1 = 1.0$.

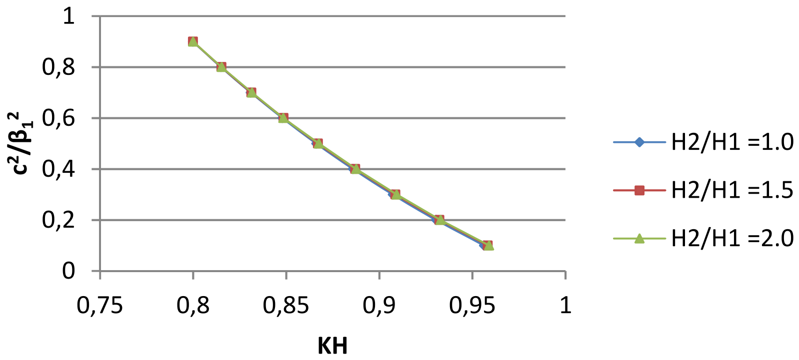

Figure 4: Variation of dimensionless phase velocity $(c/\beta_1)^2$ against dimensionless wave number KH demonstrating the influence of thickness ratio H_2/H_1 for $\alpha/2K = 1.0$, $n/K = 1.0$.

Figure 2 shows the effect of non-homogeneity parameter $\alpha/2K$ involved in the rigidity and density of the upper crustal layer. Following observations and effects are obtained under the above considered values.

(1a) For a particular dimensionless wave number KH and fixed value of non-homogeneity in rigidity and density of intermediate layer and thickness ratio i.e. $n/K=1.0$ and $H_2/H_1=1.0$, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves decreases, as the value of $\alpha/2K$ increases from 1.0 to 2.0.

(1b) For various values of $\alpha/2K$ and particular values of n/K and $H_2/H_1=1.0$, the phase velocity $(c/\beta_1)^2$ increases as the wave number decreases in all curves 1-3.

(1c) Curve 1(for $\alpha/2K=2.0$) is steeper than the curve 2 (for $\alpha/2K=1.5$), which in turn is steeper than curves 3 (for $\alpha/2K=1.0$), that reveals that the dimensionless non-homogeneity factor $\alpha/2K$ has a prominent effect on SH wave propagation.

Figure 3 shows the effect of non-homogeneity parameter n/K involved in the rigidity and density of the non-homogeneous intermediate layer. Following observations and effects are obtained under the above considered values.

(2a) For a particular dimensionless wave number KH and fixed value of non-homogeneity of upper layer i.e. $\alpha/2K = 1.0$, and fixed thickness ratio i.e. $H_2/H_1=1.0$, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves increases, as the value of n/K increases from 0.1 to 1.0.

(2b) For various values of n/K and particular values of $\alpha/2K$ and H_2/H_1, the phase velocity $(c/\beta_1)^2$ increases as the wave number decreases in all curves 1-4.

(2c) Curve 1(for $n/K=0.0$) is steeper than the curve 2 (for $n/K=0.1$), which in turn is steeper than curve 3, 4 (for $n/K=0.6$, $n/K=1.0$), that reveals that the dimensionless non-homogeneity factor n/K has a prominent effect on SH wave propagation.

(2d) All the four curves are coinciding as wave number approaching 0.7 and dimensionless phase velocity $(c/\beta_1)^2$ approaching 0.85.

Figure 4 shows the effect of thickness ratio H_2/H_1 on dimensionless phase velocity $(c/\beta_1)^2$. Following observations and effects are obtained under the above considered values.

(3a) For a particular dimensionless wave number KH and fixed value of non-homogeneity of upper and intermediate layers i.e. $\alpha/2K=1.0$ and $n/K = 1.0$, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves decreases, as the value of H_2/H_1 increases from 1.0 to 2.0.

(3b) For various values of $H_2/H_1$and particular values of n/K and $\alpha/2K$, the phase velocity $(c/\beta_1)^2$ increases as the wave number decreases in all curves 1-3.

9 CONCLUSIONS

Here, we has been studied the propagation of SH waves in double non-homogeneous crustal layers lying over an isotropic homogeneous half space. Closed form solutions have been derived separately for the displacements in non-homogeneous crustal layers and lower half-space. By using the asymptotic expansion of Whittaker's function we have derived the wave velocity equation for the SH waves in compact form. Dimensionless phase velocity $(c/\beta_1)^2$ is calculated numerically. The effect of various dimensionless elastic parameters, thickness ratio and non-homogeneity factors on the dimensionless phase velocity $(c/\beta_1)^2$ have been shown graphically. We make the following observations

1. For a particular dimensionless wave number KH and fixed value of non-homogeneity in rigidity and density of intermediate layer and thickness ratio i.e. n/K and H_2/H_1, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves decreases, as the value of $\alpha/2K$ increases.
2. For various values of $\alpha/2K$ and particular values of n/K and H_2/H_1, the phase velocity $(c/\beta_1)^2$ increases as the wave number decreases.
3. For various values of n/K and particular values of $\alpha/2K$ and H_2/H_1, the phase velocity $(c/\beta_1)^2$ increases as the wave number decreases.
4. For a particular dimensionless wave number KH and fixed value of non-homogeneity of upper layer i.e. $\alpha/2K$, and fixed thickness ratio i.e. H_2/H_1, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves increases, as the value of n/K increases.
5. For a particular dimensionless wave number KH and fixed value of non-homogeneity of upper and intermediate layers i.e. $\alpha/2K$ and $n/K = 1.0$, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves decreases, as the value of H_2/H_1 increases.
6. For various values of $H_2/H_1$and particular values of n/K and $\alpha/2K$, the phase velocity $(c/\beta_1)^2$ increases as the wave number decreases.

The study of the shear wave propagation in layered media has some possible applications in the field of earthquake engineering, seismology, civil engineering and geophysics. In geophysics, they are helpful not only in the exploration of internal structure of earth which consist of many crustal layers but also in the exploration of valuable materials buried inside the earth like minerals, metals, hydrocarbons and petroleum, etc. In earthquake engineering and seismology, Shear waves are very useful in predicting earthquakes (in-terms of finding their epicentre) in dynamic response of soils and man-made structures. SH-waves cause more destruction to the structure than the body waves due to slower attenuation of the energy. So this study may be useful for understanding the cause and estimation of damage due to earthquakes. Hence, the present theoretical study may be useful for the practical application of shear waves in the non-homogeneous layered media.

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