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Dynamic Analysis of Maneuvering Flexible Spacecraft Appendage Using Higher Order Sandwich Panel Theory

Abstract

This paper is devoted to the vibration of rotating flexible spacecraft. The maneuver of the spacecraft is modeled by a constant torque input acting on the hub. For the first time in this paper the equations of motion of flexible spacecraft are derived based on higher order sandwich panel theory (HSAPT). Hamilton's principle is used for driving the governing partial differential equations of motion. The generalized differential quadrature method (GDQ) is utilized to solve the partial differential equations of motion. The effect of different parameters on vibration of rotating flexible spacecraft appendage is investigated. It is also investigated the effect of these parameters on natural frequencies. The result of HSAPT and Euler Bernoulli theory is compared with each other. To show the accuracy the natural frequencies of recent paper are compared with the literature.

Keywords

Flexible Spacecraft, Constant torque, Hamilton's principle, GDQ, HSAPT

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1 INTRODUCTION

There are many studies on flexible spacecraft which is consisted of a rigid hub in the center and attached flexible appendages. The analysis of these systems is complicated because of coupling between rigid hub and flexible appendage. In these structures vibration of the flexible part causes the rotation of the hub and vice versa (Hu, 2009). There have been a lot of intensive researches on rotating of the flexible spacecraft and finding an appropriate method to suppress the vibrations. They usually consider each appendages as Euler-Bernoulli beam to simplify the equation of motions. Azadi et. al (Azadi, et al., 2011)studied on three axes slewing maneuver and the vibration of a flexible spacecraft. They used an adaptive-robust control scheme to achieve the satellite's large angel trajectory tracking and suppress the vibration of the appendages. They also applied an Euler-Bernoulli beam theory for flexible appendage. Maganti et. al (Maganti & Singh, 2007) present the design of a new simple adaptive system for the rotational maneuver and vibration suppression of an orbiting spacecraft with flexible appendages. They choose a control output variable as combination of pitch angle and its derivative without considering elastic mode. Tugrul Oktay (Oktay, 2015) studied on bending control of rotating Euler Bernoulli beam. He assumed that the free fixed elastic beam is attached to a servomotor using variance constrained controller, specifically output variance constrained controller for vibration suppression. Equations of motion of the system obtained via Hamilton's principle and Galerkin method.

Shahravi and azimi studied on vibration control of smart flexible sub-structure of satellite during attitude maneuver. They compared collocated and non-collocated piezoceramic patches as sensor and actuator to suppress vibration of flexible substructure. They used finite element method associated with lagrangian formulation to derive mathematical model of the slewing flexible spacecraft (Shahravi & Azimi, 2014). Shahravi and Azimi (Shahravi & Azimi, 2014) presented a new control approach for large angle attitude maneuver of flexible spacecraft. They used singular perturbation theory as a useful tool for two time rate scale separation of rigid and flexible body dynamics. The resulting slow and fast subsystems, enabling the use of two control approach for attitude (Modified Sliding Mode) and vibration Strain Rate Feedback (SRF) control of flexible spacecraft, respectively. Shahravi and Azimi (Shahravi & Azimi, 2015) studied on multi-axis attitude control of flexible spacecraft using smart structures and hybrid control scheme**.** They considered the spacecraft as a rigid hub with two elastic appendages embedded with piezoelectric sensor/actuator patches. Using a modified sliding mode control by introducing a synthesized sliding manifold ensures that the spacecraft follows the shortest possible path to the desired orientation and highly reduce the switching action and excitation of flexible modes. Zhang e. al (Zhang, et al., 2014), used PZT transducers on flexible links as vibration sensors and actuators to suppress the vibration. They derived dynamic model of the flexible links with dynamic of PZT actuators incorporated and discretized the elastic motion of the flexible links by assumed mode method. An Efficient modal control, in which the feedback forces in different modes are determined according to the vibration amplitude or energy of their own, is employed to control the PZT actuators to realize active vibration suppression.

As we know multi body systems with large deflection are being used in many practical engineering such as aeronautics, aerospace's and robotics where a flexible appendage is made of sandwich panel. A sandwich structures with soft cores made of foam or low strength honeycomb, like aramid or nomex, are being used in various industrial applications such as aerospace and civil engineering structures. Sandwich panels consist of two thin composite or thick metallic layers which is separated by a thick lightweight core. The core has different types as Honeycomb, Web core, Balsa Wood, Foamed polymer. This configuration leads to a sandwich panel with high strength and stiffness, light weight and high energy absorption capability. Because of these properties, sandwich structures are being used in aerospace equipment devices widely. It should be added that the core should be strong in shear and tension in thickness direction and must have a low density to reduce weight. There are different theories to describe the behavior of sandwich panels due to type of the core. It is common in the analysis of sandwich panels to neglect the transverse deformation of the core (Vinson, 1999). This theory is being used as the core is stiffen or rigid. Several people have investigated on these of sandwich panels. An early theory of sandwich structures, is known as the First-Order Shear

Deformation Theory (FOSDT), which is the beam/plate theory by taking into account the shear rigidity of the core, but this theory is still assumes that longitudinal deformation is linear in the thickness coordinate and the core is infinitely rigid in the transverse direction. FOSDT assumes a uniform shear stain through the height of the panel. Although this model is simple, but it is acceptable when the sandwich core is too stiff through the thickness and statically loaded. In general, and especially in modern sandwich panels, the core is flexible in all directions. There are many modified theories which considered different assumption due to better model the stress, strain, displacement distribution along the thickness. Hoh et al (Hohe, et al., 2006) investigated on the effect of the transverse compressibility of the core on the transient dynamic response of structural sandwich panels under rapid loading conditions. Frostig (Frostig, et al., 1992) is one who has done numerous studies on the behavior of sandwich panels. He offered a higher order sandwich panel theory (HSAPT) for compressible cores which in-plane stresses is neglected in it. Regardless of axial stress in the core and according to the static equilibrium equation, there will be a constant shear distribution within the core thickness and this type of approximation for the sandwich construction with a soft core would be a good approximation of the static problems. Static formulation of HSAPT is being used for many issues. Comparison between HSAPT with elasticity and experiment has shown that HSAPT predicts displacements and axial strain at the surface and near to the supports and concentrated load regions accurately. Regarding the core, though HSAPT is a good approximate theory away from supports, and concentrated load regions, it show inaccurate shear stress and axial strain through-thickness distributions adjacent to regions of concentrated loads and supports. This theory can be used for the study of composite beams and composite plates with soft cores. Frostig et. al (Frostig, et al., 2013) investigated on the free vibration response of a unidirectional sandwich panel with a compressible and incompressible core using the various computational models and it is presented and compared with the numerical investigation. Also it should be added that HSAPT theory divides into two displacement and mixed formulation theories which is different in the formulation of the problem. Frostig and baruch (Frostig & Baruch, 1994) investigated on free vibrations of sandwich beams with a transversely flexible core. The analysis presented embodies a general rigorous approach including higher order effects owing to the non-linearity of the displacement fields of the core caused by its flexibility in the vertical direction. Swanson and kim (Swanson & Kim, 2000) studied on comparison of a higher order theory for sandwich beams with finite element and elasticity analysis.

There are different methods to solve these problems. Damanpack and khalili (Damanpack & Khalili, 2012) is investigated on the higher-order free vibration of sandwich beams with a flexible core using dynamic stiffness method. One of the methods to solve the vibration analysis is generalized differential quadrature (GDQ) which is used in different papers. There are a lot of people which used this method to analysis the problems. Tornabene and Viola (Tornabene & Viola, 2009) studied the dynamic behavior of functionally graded parabolic and circular panels and shells of revolution. They used First-order Shear Deformation Theory (FSDT) to study the moderately thick structural elements. They also investigated on the local GDQ method applied to general higher-order theories of doubly-curved laminated composite shells and panels (Tornabene & Viola, 2014). Hong et al. investigated on thermal induced vibration of a thermal sleeve with GDQ method. They used this method to obtain the numerical results of two-layer cross-ply laminated tubes under thermal vibration (Hong, et al., 2005).

The present paper provides an analysis to obtain the response of a flexible spacecraft with respect to a constant torque on the hub for a real sample flexible spacecraft. Unlike the previous papers the theory which is applied for flexible spacecraft is HSAPT and the result for vibration analysis is compared with other conventional beams theory.

2 DERIVATION OF THE GOVERNING EQUATIONS

Figure 1 shows the model of a flexible spacecraft, which consists of a rigid hub with radius a, and a cantilever sandwich beam with length L. The sandwich beam with width b is made of three layers. Two thin stiff face sheets with thickness h_f and a thick soft core with thickness h_c.

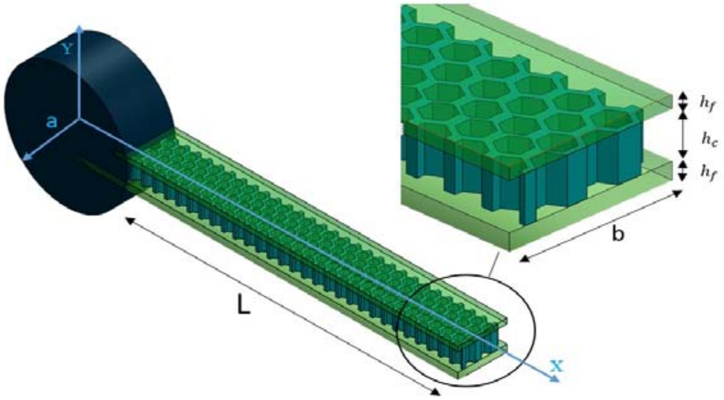

Figure 1: Geometry of rigid hub with flexible appendage.

The general assumption for derivation of general equation of motion for a sandwich beam is as follow

1. All the deformations and strains are very small and the theory of linear elasticity is applied.
2. The face sheets and the core of the beam are made of isotropic and homogeneous materials. The sandwich beam is assumed to be symmetric.
3. Transverse normal strains are negligible in the face sheets.
4. There is no slippage of delamination between the layers.

2.1 Displacement Theory

The face sheets deform as Bernoulli-Euler beam theory and the normal stress of the core is negligible and it is soft through the thickness. Here the axial and transverse displacements of the top face sheet are $u_t(x,t), w_t(x,t)$ and the transverse and axial displacements of the bottom face sheets are $u_b(x,t), w_b(x,t)$, respectively. The axial and transverse displacements of the core layer are $u_c(x,t), w_c(x,t)$ respectively.

The displacement filed for a sandwich beam assumes as

$$
X: \quad \begin{cases} \text{Tope face} & u_t - y w_t' & \frac{h_c}{2} \langle y \langle \frac{h_c}{2} + h_f \\ \text{Core} & u_c + \varphi_1 y + \varphi_2 y^2 + \varphi_3 y^3 & \frac{-h_c}{2} \langle y \langle \frac{h_c}{2} \\ \text{Bottom Face} & u_b - y w_b' & \frac{-h_c}{2} - h_f \langle y \langle -\frac{h_c}{2} \end{cases} \tag{1}
$$

$$
Y: \quad \begin{cases} \text{Tope face} & w_t & \frac{h_c}{2} \langle y \langle \frac{h_c}{2} + h_f \\ \text{Core} & w_c + \psi_1 y + \psi_2 y^2 & \frac{-h_c}{2} \langle y \langle \frac{h_c}{2} \\ \text{Bottom Face} & w_b & \frac{-h_c}{2} - h_f \langle y \langle -\frac{h_c}{2} \end{cases}
$$

By using the compatibility conditions as

$$
\begin{cases} X^{Core}(y = \frac{h_c}{2}) = X^{Topface}(y = \frac{h_c}{2}) \\ X^{Core}(y = -\frac{h_c}{2}) = X^{Bottomface}(y = -\frac{h_c}{2}) \end{cases}
\quad
\begin{cases} Y^{Core}(y = \frac{h_c}{2}) = Y^{Topface}(y = \frac{h_c}{2}) \\ Y^{Core}(y = -\frac{h_c}{2}) = Y^{Bottomface}(y = -\frac{h_c}{2}) \end{cases} \tag{2}
$$

The displacement fields (Equation 1) change as following

$$
X: \quad \begin{cases} \text{Tope face} & u_t - y w_t' & \frac{h_c}{2} \langle y \langle \frac{h_c}{2} + h_f \\ \text{Core} & u_c + \varphi_1 y + (u_t + u_b + \frac{h_f}{2} w_t' - \frac{h_f}{2} w_b' - 2u_c)\frac{2y^2}{h_c^2} + (u_t - u_b + \frac{h_f}{2} w_t' + \frac{h_f}{2} w_b' - h_c \varphi_1)\frac{4y^3}{h_c^3} & \frac{-h_c}{2} \langle y \langle \frac{h_c}{2} \\ \text{Bottom Face} & u_b - y w_b' & \frac{-h_c}{2} - h_f \langle y \langle -\frac{h_c}{2} \end{cases} \tag{3}
$$

$$Y:\quad \begin{cases} w_t & \text{Tope face} \quad \frac{h_c}{2} \langle y \langle \frac{h_c}{2} + h_f \\ w_c + (w_t - w_b)\, y/h_c + (w_t + w_b - 2w_c)\frac{2y^2}{h_c^{\,2}} & \text{Core} \quad \frac{-h_c}{2} \langle y \langle \frac{h_c}{2} \\ w_b & \text{Bottom Face} \quad \frac{-h_c}{2} - h_f \langle y \langle -\frac{h_c}{2} \end{cases}$$

Because the hub is rigid therefore the total potential energy is just for sandwich panel

$$\mathrm{U} = \frac{1}{2}\{\int_{v_t} \sigma_x \varepsilon_x dV_t + \int_{v_c} (\sigma_y \varepsilon_y + \tau_{xy}\gamma_{xy}) dV_c + \int_{v_b} \sigma_x \varepsilon_x dV_b\}$$
$$\mathrm{U} = \frac{1}{2}\{\int_{v_t} E_f \varepsilon_x^{\,2} dV_t + \int_{v_c} (E_c \varepsilon_y^{\,2} + G_c \gamma_{xy}^{\,2}) dV_c + \int_{v_b} E_f \varepsilon_x^{\,2} dV_b\} \tag{4}$$

Here E_f, E_c are modulus of elasticity of face sheets and the core respectively and G_c is the shear modulus of the core. By substituting the strain displacement relations (3) in potential energy function (4)

$$U = \frac{1}{2}\int_0^l \{\int_{A_t} E_f (u_t' - y w_t')^2 dA_t + \int_{A_b} E_f (u_b' - y w_b')^2 dA_b$$
$$+ \int_{A_c} E_c[(w_t - w_b)\frac{1}{h_c} + (w_t + w_b - 2w_c)\frac{4y}{h_c^2}]^2 dA_c\}$$
$$+ \frac{1}{2}\int_0^l \int_{A_c} G_c[(\varphi_1 - w_c') + (u_t + u_b + \frac{2h_f + h_c}{4} w_t' - \frac{2h_f + h_c}{4} w_b' - 2u_c)\frac{4y}{h_c^2}]\, dA_c \tag{5}$$
$$+ \frac{1}{2}\int_{A_c} (u_t - u_b + \frac{3h_f + h_c}{6} w_t' + \frac{3h_f + h_c}{6} w_b' - h_c\varphi_1 - \frac{h_c}{3} w_c')\frac{12y^2}{h_c^3}]^2 dA_c\} dx$$

The kinetic energy for flexible satellite is as

$$T = \frac{1}{2}\{\int_{V_t} \rho_f (\dot{X}^2 + \dot{Y}^2)\, dV_t + \int_{V_b} \rho_f (\dot{X}^2 + \dot{Y}^2)\, dV_b + \int_{V_c} \rho_c (\dot{X}^2 + \dot{Y}^2)\, dV_c\} + \frac{1}{2} J_{hub}\dot{\theta}^2\}$$
$$T = \frac{1}{2}\{\int_{V_t} \rho_f (\dot{x}_t^{\,2} + (\dot{y}_t + (x+r)\dot{\theta})^2)\, dV_t + \int_{V_b} \rho_f (\dot{x}_b^{\,2} + (\dot{y}_b + (x+r)\dot{\theta})^2)\, dV_b \tag{6}$$
$$+ \int_{V_c} \rho_c (\dot{x}_c^{\,2} + (\dot{y}_c + (x+r)\dot{\theta})^2)\, dV_c\} + \frac{1}{2} J_{hub}\dot{\theta}^2$$

Here ρ_f is the face sheets density, ρ_c is the core density, r is the hub radius, and J_{hub} is moment inertia of the rigid hub. The energy of external applied torque is as

$$F = M.\theta \tag{7}$$

Here M is applied external Torque on the hub. By substituting the relation for displacement into energy function and using the Hamilton principle the governing partial differential equations of motion and boundary conditions are being obtained.

$$\begin{aligned} & m_f(\ddot{u}_t) - m_f(\frac{h_f+h_c}{2})\ddot{w}'_t + m_c(\frac{1}{20}(\ddot{u}_t+\ddot{u}_b) + \frac{1}{28}(\ddot{u}_t-\ddot{u}_b) + \frac{1}{15}\ddot{u}_c + \frac{1}{70}h_c(\ddot{\phi}+\ddot{\theta}) \\ & + \frac{1}{40}h_f(\ddot{w}'_t-\ddot{w}'_b) + \frac{1}{56}h_f(\ddot{w}'_t-\ddot{w}'_b) - E_fA_fu''_t + E_fA_f(\frac{h_f+h_c}{2})w'''_t + \frac{G_cA_c}{h_c^2}(\frac{4}{3}(u_t+u_b) \\ & + \frac{9}{5}(u_t-u_b) - \frac{8}{3}u_c - \frac{4}{5}h_c\phi + (\frac{2}{3}h_f + \frac{1}{3}h_c)(w'_t+w'_b) + (\frac{9}{10}h_f + \frac{3}{10}h_c)(w'_t+w'_b) + \frac{2}{5}h_cw'_c) = 0 \end{aligned} \tag{8}$$

$$\begin{aligned} & m_f(\ddot{u}_b) - m_f(\frac{h_f+h_c}{2})\ddot{w}'_b + m_c(\frac{1}{20}(\ddot{u}_t+\ddot{u}_b) - \frac{1}{28}(\ddot{u}_t-\ddot{u}_b) + \frac{1}{15}\ddot{u}_c - \frac{1}{70}h_c(\ddot{\phi}+\ddot{\theta}) \\ & + \frac{1}{40}h_f(\ddot{w}'_t-\ddot{w}'_b) - \frac{1}{56}h_f(\ddot{w}'_t-\ddot{w}'_b)) - E_fA_fu''_b + E_fA_f(\frac{h_f+h_c}{2})w'''_b + \frac{G_cA_c}{h_c^2}(\frac{4}{3}(u_t+u_b) \\ & - \frac{9}{5}(u_t-u_b) - \frac{8}{3}u_c + \frac{4}{5}h_c\phi + (\frac{2}{3}h_f + \frac{1}{3}h_c)(w'_t-w'_b) - (\frac{9}{10}h_f - \frac{3}{10}h_c)(w'_t+w'_b) - \frac{2}{5}h_cw'_c) = 0 \end{aligned} \tag{9}$$

$$\begin{aligned} & m_f(\ddot{w}_t - (\frac{h_c+h_f}{2})\ddot{u}'_t + (\frac{h_c^2}{4} + \frac{h_ch_f}{2} + \frac{h_f^2}{3})\ddot{w}''_t + (r+\frac{l}{2})\ddot{\theta}) + m_c(-\frac{1}{40}h_f(\ddot{u}'_t+\ddot{u}'_b) - \frac{1}{56}h_f(\ddot{u}'_t+\ddot{u}'_b) \\ & -\frac{1}{30}h_f\ddot{u}'_c - \frac{1}{140}h_fh_c\ddot{\varphi}' - \frac{1}{80}h_f^2(\ddot{w}''_t-\ddot{w}''_b) + \frac{1}{12}(\ddot{w}_t-\ddot{w}_b) - \frac{1}{112}h_f^2(\ddot{w}''_t+\ddot{w}''_b) + \frac{1}{20}(\ddot{w}_t+\ddot{w}_b) \\ & + \frac{1}{15}\ddot{w}_c + (\frac{1}{10}+\frac{1}{15})(r+\frac{l}{2})\ddot{\theta}) + E_fA_f[(\frac{h_c^2}{4} + \frac{h_ch_f}{2} + \frac{h_f^2}{3})w''''_t - (\frac{h_c+h_f}{2})u'''_t] \\ & + \frac{GA_c}{h_c^2}[-(\frac{2}{3}h_f + \frac{1}{3}h_c)(u'_t+u'_b) - (\frac{9}{10}h_f + \frac{3}{10}h_c)(u'_t-u'_b) + (\frac{4}{3}h_f + \frac{2}{3}h_c)u'_c \\ & + (\frac{2}{5}h_fh_c + \frac{2}{15}h_c^2)(\varphi') - (\frac{1}{3}h_fh_c + \frac{1}{3}h_f^2 + \frac{1}{12}h_c^2)(w''_t-w''_b) - (\frac{9}{20}h_f^2 + \frac{3}{10}h_fh_c + \frac{1}{20}h_c^2) \\ & (w''_t+w''_b) - (\frac{1}{5}h_fh_c + \frac{1}{15}h_c^2)w''_c] + \frac{E_cA_c}{h_c^2}[\frac{4}{3}(w_t+w_b) + (w_t-w_b) - \frac{8}{3}w_c] = 0 \end{aligned} \tag{10}$$

$$
\begin{aligned}
&m_f(\ddot{w}_b-(\frac{h_c+h_f}{2})\ddot{u}_b'+(\frac{h_c^2}{4}+\frac{h_ch_f}{2}\frac{h_f^2}{3})\ddot{w}_b''+(r+\frac{l}{2})\ddot{\theta})+m_c(\frac{1}{40}h_f(\ddot{u}_t'+\ddot{u}_b')-\frac{1}{56}h_f(\ddot{u}_t'+\ddot{u}_b')+\frac{1}{30}h_f\ddot{u}_c') \\
&-\frac{1}{140}h_fh_c\ddot{\varphi}'+\frac{1}{80}h_f^{\,2}(\ddot{w}_t''-\ddot{w}_b'')-\frac{1}{12}(\ddot{w}_t-\ddot{w}_b)-\frac{1}{112}h_f^{\,2}(\ddot{w}_t''+\ddot{w}_b'')+\frac{1}{20}(\ddot{w}_t+\ddot{w}_b)+ \\
&\frac{1}{15}\ddot{w}_c+(\frac{1}{10}+\frac{1}{15})(r+\frac{l}{2})\ddot{\theta})++E_fA_f[(\frac{h_c^2}{4}+\frac{h_ch_f}{2}+\frac{h_f^2}{3})w_b''''-(\frac{h_c+h_f}{2})u_b'''] \\
&+\frac{GA_c}{h_c^{\,2}}[(\frac{2}{3}h_f+\frac{1}{3}h_c)(u_t'+u_b')-(\frac{9}{10}h_f+\frac{3}{10}h_c)(u_t'-u_b')-(\frac{4}{3}h_f+\frac{2}{3}h_c)u_c' \\
&+(\frac{2}{5}h_fh_c+\frac{2}{15}h_c^2)(\varphi'')+(\frac{1}{3}h_fh_c+\frac{1}{3}h_f^{\,2}+\frac{1}{12}h_c^{\,2})(w_t''-w_b'')-(\frac{9}{20}h_f^{\,2}+\frac{3}{10}h_fh_c+\frac{1}{20}h_c^{\,2})(w_t''+w_b'') \\
&-(\frac{1}{5}h_fh_c+\frac{1}{15}h_c^2)w_c'']+\frac{E_cA_c}{h_c^2}[\frac{4}{3}(w_t+w_b)-(w_t-w_b)-\frac{8}{3}w_c]=0
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
&m_c(\frac{8}{15}\ddot{u}_c+\frac{1}{15}(\ddot{u}_b+\ddot{u}_t)+\frac{1}{30}h_f(w_t'-w_b') \\
&+\frac{G_cA_c}{h_c^2}[-\frac{8}{3}(u_t+u_b)+\frac{16}{3}u_c-(\frac{4}{3}h_f+\frac{2}{3}h_c)(w_t'-w_b')]=0
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
&m_c(\frac{2}{105}h_c^2(\ddot{\phi}+\ddot{\theta})+\frac{1}{70}h_c(\ddot{u}_t+\ddot{u}_b)+\frac{1}{140}h_fh_c(\ddot{w}_t'+\ddot{w}_b') \\
&+\frac{G_cA_c}{h_c^2}[-\frac{4}{5}h_c(u_t-u_b)+\frac{4}{5}h_c^2\phi-(\frac{2}{15}h_c^2+\frac{2}{5}h_ch_f)(w_t'+w_b')+\frac{4}{15}h_c^2w_c']=0
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
&m_c(\frac{8}{15}\ddot{w}_c+\frac{1}{15}h_c(\ddot{w}_b+\ddot{w}_t)+(\frac{8}{15}+\frac{2}{15})(r+\frac{l}{2})\ddot{\theta})+\frac{G_cA_c}{h_c^2}[-\frac{2}{5}h_c(u_t'-u_b')-\frac{4}{15}h_c^{\,2}\phi' \\
&-(\frac{1}{5}h_fh_c+\frac{1}{15}h_c^2)(w_t''+w_b'')-\frac{8}{15}h_c^2w_c'']+\frac{E_cA_c}{h_c^2}[-\frac{8}{3}(w_t+w_b)+\frac{16}{3}w_c]=0
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
&\frac{1}{70}m_ch_c\ddot{u}_t-\frac{1}{70}m_ch_c\ddot{u}_b+(m_f+\frac{m_c}{6})(r+\frac{l}{2})\ddot{w}_t+\frac{1}{140}m_ch_ch_f\ddot{w}_t'+\frac{1}{140}m_ch_ch_f\ddot{w}_b' \\
&+\frac{2}{3}m_c(r+\frac{l}{2})\ddot{w}_c+\frac{2}{105}m_ch_c^2\ddot{\phi}+\frac{2}{105}m_ch_c^2\ddot{\theta}+(2m_f+m_c)(r^2+\frac{l^2}{3}+rl)\ddot{\theta}+J_h\ddot{\theta}=M
\end{aligned}
\tag{15}
$$

The axial forces $P_{t,b}(x,t)$, the shear forces $V_{t,b,c}(x,t)$ and the bending moments$M_{t,b}(x,t)$:

$$
\begin{aligned}
P_t &= -E_fA_f(\frac{\partial u_t}{\partial x}-(\frac{h_c+h_f}{2})\frac{\partial^2 w_t}{\partial x^2}) \\
P_b &= -E_fA_f(\frac{\partial u_b}{\partial x}-(\frac{h_c+h_f}{2})\frac{\partial^2 w_b}{\partial x^2})
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
V_t &= -m_f\left(\left(\frac{h_f^2}{3}+\frac{h_c^2}{4}+\frac{h_f h_c}{2}\right)\ddot{w}_t' - \left(\frac{h_f+h_c}{2}\right)\ddot{u}_t\right) \\
&+\frac{m_c}{840}\left(-36h_f\ddot{u}_t - 6h_f\ddot{u}_b - 28h_f\ddot{u}_c - 6h_f h_c\ddot{\varphi} + 3h_f^2\ddot{w}_b' - 18h_f^2\ddot{w}_t'\right) \\
&+E_f A_f\left(\left(\frac{h_c^2}{4}+\frac{h_f h_c}{2}+\frac{h_f^2}{3}\right)w_t''' - \left(\frac{h_f+h_c}{2}\right)u_t''\right) \\
&+\frac{G_c A_c}{840h_c^2}[(196h_f - 28h_c)u_b - (1316h_f + 532h_c)u_t + (560h_c + 1120h_f)u_c \\
&+(28h_c^2 - 98h_f^2 + 28h_f h_c)w_b'' - (112h_c^2 + 658h_f^2 + 532h_f{}^2 + 532h_f h_c)w_t' \\
&+(336h_f h_c + 112h_c^2)\varphi - (56h_c^2 + 168h_f h_c)w_c']
\end{aligned}
$$

$$
\begin{aligned}
V_b &= -m_f\left(\left(\frac{h_f^2}{3}+\frac{h_c^2}{4}+\frac{h_f h_c}{2}\right)\ddot{w}_b' - \left(\frac{h_f+h_c}{2}\right)\ddot{u}_b\right) \\
&+\frac{m_c}{840}\left(6h_f\ddot{u}_t + 36h_f + 28h_f\ddot{u}_c - 6h_f h_c\ddot{\varphi} + 3h_f^2\ddot{w}_t' - 18h_f^2\ddot{w}_b'\right) \\
&+E_f A_f\left(\left(\frac{h_c^2}{4}+\frac{h_f h_c}{2}+\frac{h_f^2}{3}\right)w_b''' - \left(\frac{h_f+h_c}{2}\right)u_b''\right) \\
&+\frac{G_c A_c}{840h_c^2}[(-196h_f + 28h_c)u_t + (1316h_f + 532h_c)u_b - (560h_c + 1120h_f)u_c \\
&+(28h_c^2 - 98h_f^2 + 28h_f h_c)w_t'' - (112h_c^2 + 658h_f^2 + 532h_f{}^2 + 532h_f h_c)w_b' \\
&+(336h_f h_c + 112h_c^2)\varphi - (56h_c^2 + 168h_f h_c)w_c']
\end{aligned}
$$

$$
V_c = \frac{G_c A_c}{840h_c^2}[-336h_f(u_t - u_b)_c - (224h_c^2)\varphi - (56h_c^2 + 168h_f h_c)(w_t' - w_b') - 448h_c^2 w_c']
$$

$$
M_t = -E_f A_f\left(\left(\frac{h_c^2}{4}+\frac{h_f h_c}{2}+\frac{h_f^2}{3}\right)w_t'' - \left(\frac{h_f+h_c}{2}\right)u_t'\right)
$$

$$
M_b = -E_f A_f\left(\left(\frac{h_c^2}{4}+\frac{h_f h_c}{2}+\frac{h_f^2}{3}\right)w_b'' - \left(\frac{h_f+h_c}{2}\right)u_b'\right)
$$

The boundary conditions for a sandwich beam:

$$\begin{aligned}
&\delta u_t(0,t)=0, P_t(l,t)=0\\
&\delta w_t(0,t)=0, V_t(l,t)=0\\
&\delta w_t^{'}(0,t)=0, M_t(l,t)=0\\
&\delta w_c(0,t)=0, V_c(l,t)=0\\
&\delta u_b(0,t)=0, P_b(l,t)=0\\
&\delta w_b(0,t)=0, V_b(l,t)=0\\
&\delta w_b^{'}(0,t)=0, M_b(l,t)=0
\end{aligned} \tag{17}$$

3 NUMERICAL RESULTS AND DISCUSSION

There are different arrangements of grid points which are being used in GDQ method. Equal spacing sample of grid points used in earlier papers give some inaccurate results. Here the Chebyshev-Gauss-Lobatto distribution is utilized to discretize the spatial domain as follows (Zong & Zhang, 2009)

$$x_i=\frac{l}{2}[1-\cos(\frac{\pi(i-1)}{N-1})] \qquad \text{and} \qquad i=1,...,N \tag{18}$$

Natural frequencies (rad/s)	Present paper	Ref. (Damanpack & Khalili, 2012)	Ref. (Sokolinsky & Nutt, 2004)
1	1031.02	1030.962	1036.725
2	3195.114	3194.948	3210.708
3	5671.588	5617.640	5717.698
4	8517.974	8517.640	8658.229
5	11910.109	11909.750	12189.379
10	15945.013	15944.648	16411.680

Table 1: The lowest natural frequencies for a cantilever sandwich beam (Antisymmetric Modes).

Natural frequencies (rad/s)	Present paper	Ref. (Damanpack & Khalili, 2012)	Ref. (Sokolinsky & Nutt, 2004)
6	14473.978	14473.978	15029.379
7	14490.414	14490.419	15048.228
8	14500.200	14664.758	15236.724
9	15248.520	14664.739	15858.759

Table 2: The lowest natural frequencies for a cantilever sandwich beam (Symmetric Modes).

Because in vibration of a flexible structure natural frequencies are so important in this analysis, first it should be found accurate amounts for natural frequencies. If there is no awareness of exact amount of natural frequencies and it is applied a harmonic load with frequency near to the structure's natural frequency it might lead to the colapse of structure. Therefore finding exact natural frequencies is vital in vibration analysis of structures. To examine the accuracy of the present paper it is compared the natural frequencies of recent paper with the literatures in Tables 1 and 2 and it can be seen a good accuracy of recent analysis with respect to literatures. In figures 2 three diffrenet shape modes of sandwich beam with GDQ method are also compared with Damanpak and Khalili (Damanpack & khalili, 2012) for assuraness of the results. As it can be seen in seventh shape mode which is a symmetric one, the core deformes more than the asymmetric ones.

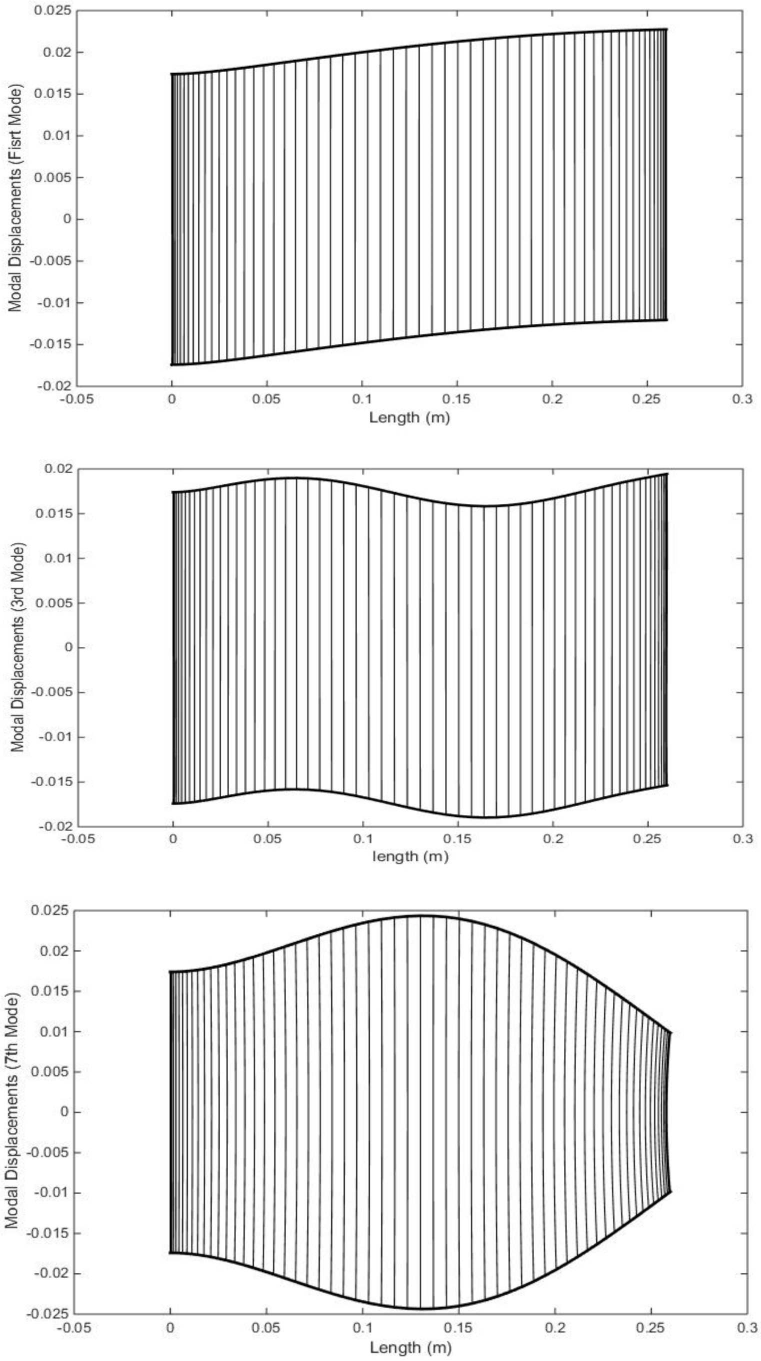

Figure 2: Shape modes of flexible sandwich appendage.

3.1 Applied Torque on the Hub

In this section it is investigated on vibration behaviors of the flexible spacecraft with physical parameters presented in table 4 for a constant applied torque on the hub($P_{hub} = 10\ N.m$).

	Parameters	
Face sheets	Honeycomb Core	Hub
Young's Modulus/Shear Modulus (GPa)	$E_f = 70$	$G_c = 0.26$
Density $\left(\frac{\text{kg}}{\text{m}^3}\right)$	$\rho_f = 2700$	$\rho_c = 83.3$
Thickness (m)	$h_f = 0.0005$	$h_c = 0.02$
Width (m)	b=0.5	b=0.5
Length (m)	$L_{t,b} = 2$	$L_c = 2$
Hub dimension (m)		a=0.3
Hub Inertia (kgm^2)		$J_h = 27.2$

Table 3: Parameters of flexible spacecraft.

Figure 3 shows that by applying a constant torque on the hub the rotation of the hub changes as parabolic with respect to time which is in comply with the mathematic of the problem. For more clarification it shoud be added that, it is clear that the torque is a kind of acceleration $(\text{Torque} \cong (\text{acceleration})\ddot{\theta} = \text{Constant} = \text{C})$ and by one time integrating of a constant acceleration, linear velocity $(\dot{\theta} = \int_0^t Cdt = \text{C}t)$ and two times integrating of that a parabolic rotation will be concluded $(\theta = \int_0^t Ctdt = \text{C}t^2 + d)$. This rotation in figure 4 give us information about the behavior of hub and satellite and how it's direction with respect to earth changes. By having this information about the rotation of hub (satellite) with respect to time, and by using an appropriate control algurithm it is possible to fix the sattelite in a special direction for sending singal to the earth.

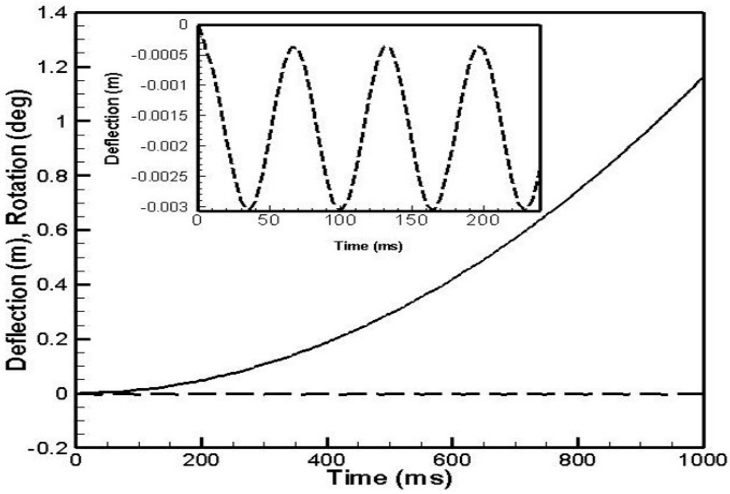

Figure 3: Rotation of the hub and tip deflection of flexible appendage.

As it can be seen in figure 3 that the deflection of the beam is up to 0.003 (m) and this deflection is logical with respect to the 0.021 (m) thickness of the beam.

Figure 4 shows lateral deflection of a rotating appendage for three different types of beam; an ordinary beam made of just face sheet's material (stiff beam), an ordinary beam made of just core's material (soft material), and a sandwich beam. But the physical dimention for three cases is equal. It can be seen that deflection of flexible appendage for sandwich beam is less than stiff and soft ones. This behavior is being related to parameter EI. The comparison of modulus of elasticity for three different kinds of materials is as $E_{Soft} < E_{Sandwich} < \mathrm{E_{Stiff}}$. Modulus of elasticity for stiff beam is more than two other cases. Comparison of moment of inertia is as$I_{Stiff} = I_{Soft} < I_{Sandwich}$. Because in sandwich beam the natural axis of stiffen faces is far from the natural axis of core therefore moment of inertia for sandwich beam is more than the two other cases with just on layer (stiff beam or soft beam) and therefore the moment of inertia for sandwich beam is much more than the amount of this parameter for stiffen beam with the same geometry. By multiplication of modulus of elasticity (E) and moment of inertia (I) and by considering the physical parameters from table 3 this comparison changes as $E_{Soft}I_{Soft} < E_{Stiff}I_{Stiff} < E_{San}I_{San}$ .Whatever this product is smaller the deflection is more. Therefore although sandwich panels are softer than stiff beams but their moment of inertia is significantly more than the moment of inertia of stiff beams and as a result this production (EI) for sandwich beam is more than two other cases, and therefore the deflection of sandwich panel with lower weight is less than a stiff beam and this is one of the advantages of sandwich structures. Figures 5-7 show the behavior of a hub appendage system for different thicknesses and widths of the sandwich beam. As it is mentioned above there are two important parameters which are effective on deflection of sandwich structures. Stiffness (E_{San}) and moment inertia of sandwich beam (I_{San}). As it is clear the stiffness of the core is less than the face sheet's. Therefore by increasing the core thickness we are increasing the proportion of softer part and therefore it is expected more deflection. But it can be seen different behavior in figure 5 that by increasing the core thickness the sandwich beam tends to lower deflection. This behavior depends on other parameter which is called moment of inertia (I_{San}). In this figure it can be seen that by increasing the core thickness we are increasing the distance of face sheets with respect to core's natural axis. This increases the moment of inertia (I_{San}) of sandwich panel and therefore the production of these two parameters ($E_{San}I_{San}$) increases. In sandwich panels the face sheets are so thin and as it is mentioned before moment of inertia parameter (I) is depend on the distance between natural axis of face sheets and the natural axis of the core. Because facesheets are so thin increasing the thickness of these parts don't have a significant effect on moment of inertia (I_{San}), but it increases the proportion of stiff part with respect to softer part (core) ($E_{San}\ increases$) and therefore sandwich beam's deformation decreases. It can be concluded that there are two ways for decreasing the deflection of the sandwich beam. First by increasing the thickness of thick soft core (increasing the moment of inertia I), and second increasing the thin stiff layers (increasing the stiffness E).

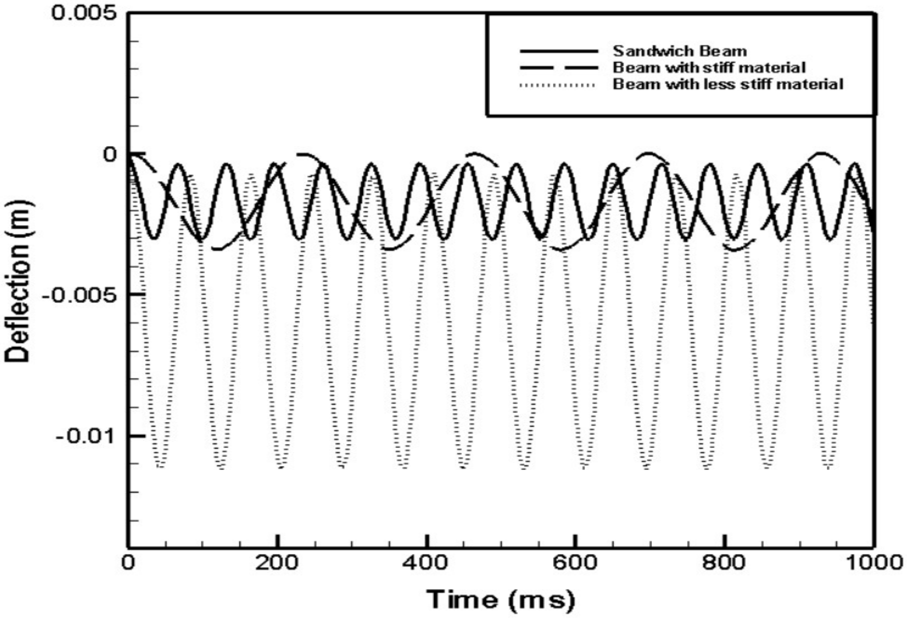

Figure 4: Tip deflection of flexible appendage for different types of beam.

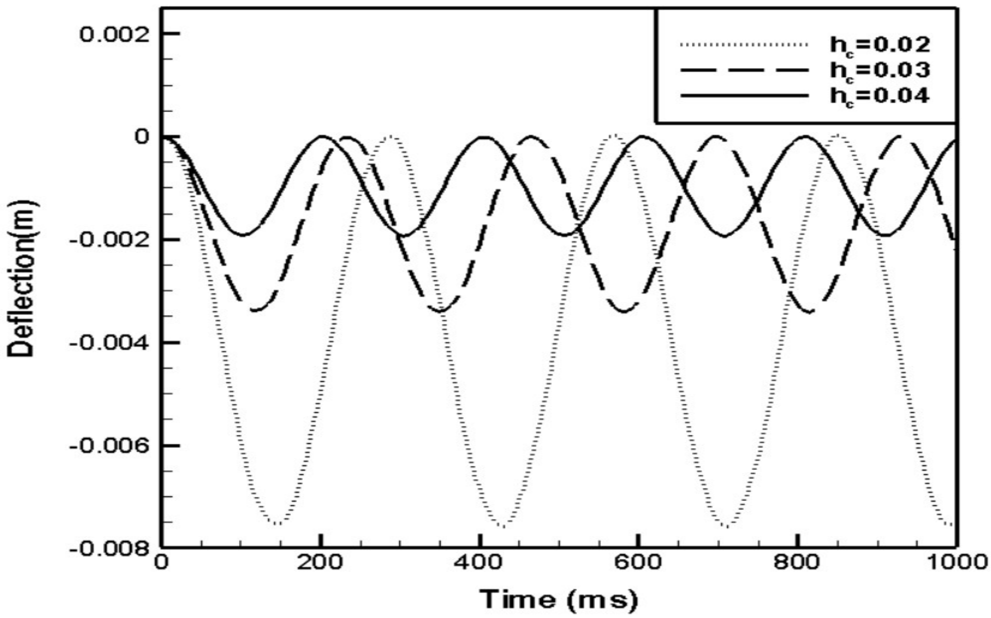

Figure 5: Tip deflection of flexible appendage for three different core thiknesses.

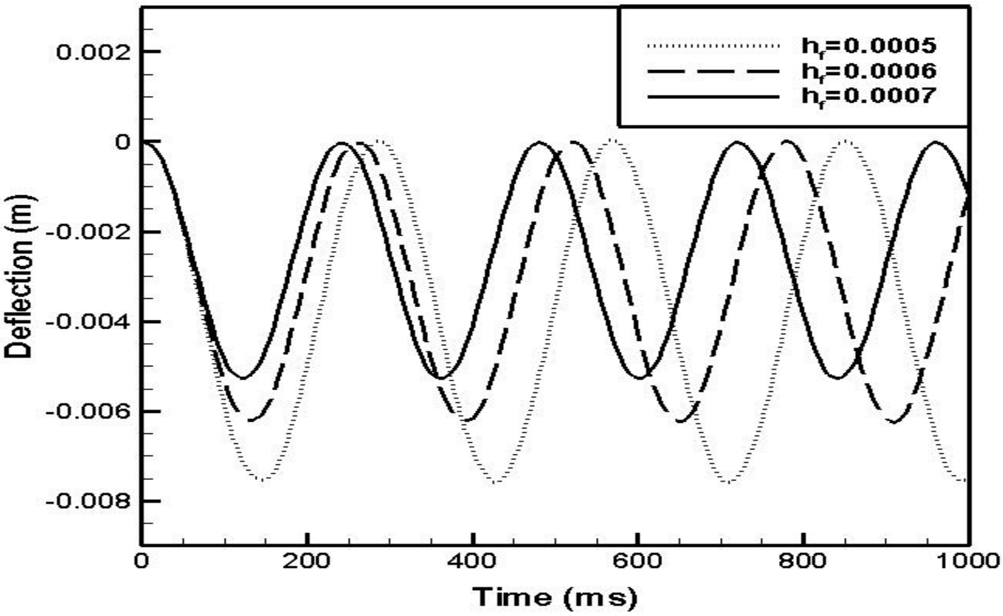

Figure 6: Tip deflection of flexible appendage for three different face sheet thicknesses.

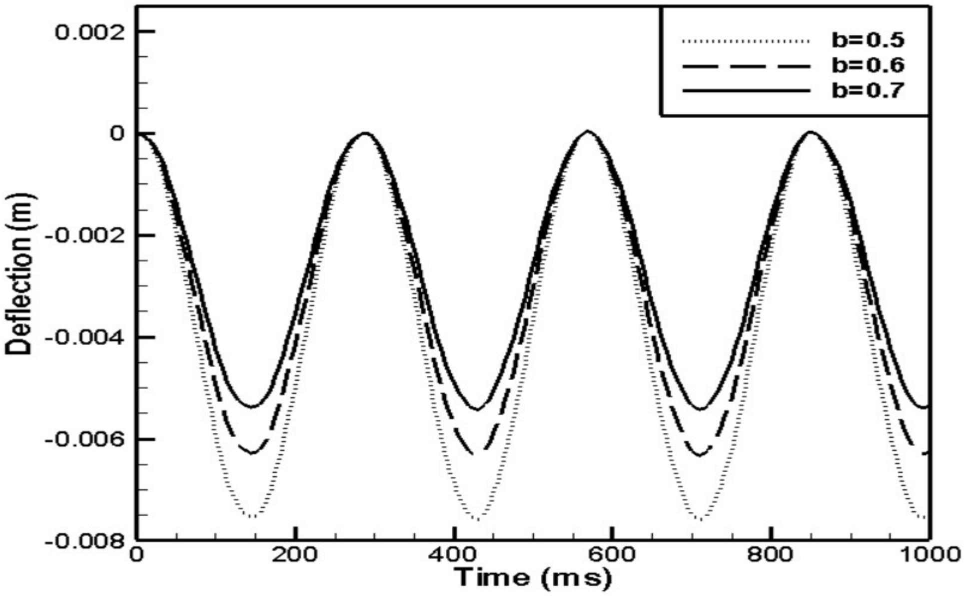

Figure 7: Tip deflection of flexible appendage for three different widths of the beam.

Figure 6 shows that by increasing the sandwich panel's width the deflection will decrease. By comparing figures 5-7 it can be seen the difference of sandwich beam's behavior with respect to variation of thickness and width of the beam. In figures 5 and 6 it can be seen that by increasing the thickness of face sheets or core thickness of the beam the frequency will change too. The natural frequency for a lateral vibration of beam is as $\omega = \beta^2 \sqrt{\frac{EI}{\rho A}} = (\beta l)^2 \sqrt{\frac{EI}{\rho A l^4}} = (\beta l)^2 \sqrt{\frac{E(\frac{1}{12} b h^3)}{\rho b h l^4}}$ (Rao,, n.d.). As it can be seen from this relation the frequency will increase by the factor $\sqrt{\frac{h^3}{h}}$ but it remains constant with respect to width of the beam (b). These behaviors can be seen in figure 5-7. As it is mentioned in previous sections, in this paper the HSAPT method is used for the first time for flexible part of the satellite instead of using Euler Bernoulli theory. Therefore here in Figure 8 it is compared these two deformation theories for flexible appendage. It can be seen that the deflection of sandwich beam for HSAPT is more than the Euler Bernoulli. Because in HSAPT it is concerened different parameters such as Shear strain and rotation, therefore the energy function here is more than the Euler Bernoulli theroy and this results in more deflection for HSAPT as it can be seen in figure 8.

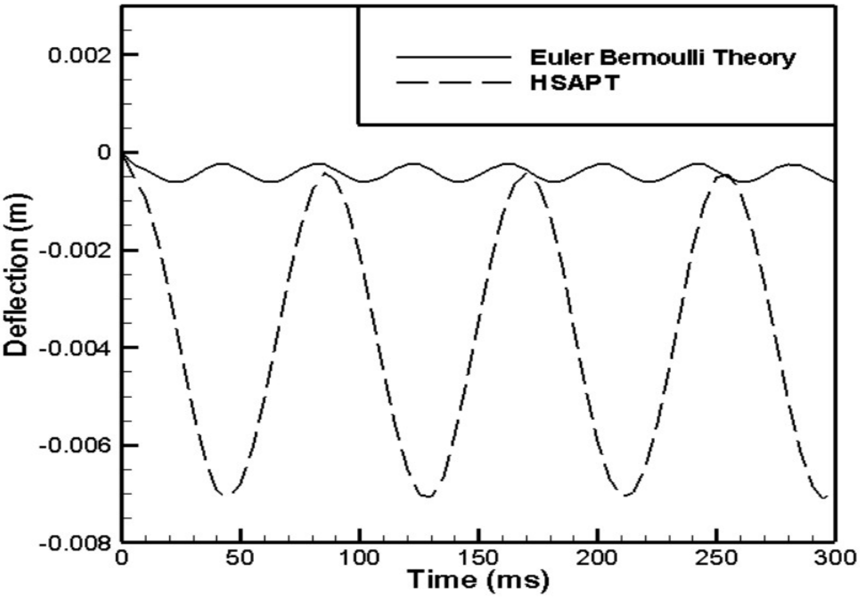

Figure 8: Tip deflection of flexible appendage for HSAPT and euler bernulli theory.

4 CONCLUSION

In the present article, the energy method is used for the vibration analysis of rotating flexible spacecraft made of sandwich beam. The governing equations of motion are derived using Hamilton principle. The GDQ method is developed to transfer the governing partial differential equations to the algebraic equations. To certify the accuracy of the present formulations, natural frequencies obtained by the present analysis are compared with the literatures. It is presented three different shape modes for flexible appendage in which can be seen a good accordance with the literatures. Finally the effect of different geometrical parameters on vibration of flexible appendage are studied. Several salient points should be mentioned here:

1. The accuracy of GDQ method with respect to other numerical methods can be discovered by comparison of the natural frequencies of this paper with the literatures.
2. The effect of compressible core in sandwich panel structure can be seen in higher frequencies and symmetric shape modes. In these frequencies the core sustains the significant part of deformation.
3. The deflection of the sandwich beam is affected by the important parameter EI. Whatever this product is more the deflection is smaller.
4. Although increasing the core thickness of sandwich beam, increases the proportional of flexible part but it can be seen that the tip deflection decreases. Actually this behavior returns to the effect of moment of inertia (I) on the tip deflection of the beam. Because we are increasing the distance between face sheets and natural axis by increasing the core thickness. As this parameter is more the moment of inertia is more and as a consequent the deflection of sandwich beam decreases.
5. Since the face sheets of sandwich structures are so thin therefore increasing their thickness hasn't significant effect on the moment of inertia of sandwich beam. But we are actually increasing the proportion of stiff part of the beam by increasing the face sheet's thickness and this decreases the tip deflection of the beam.
6. Increasing the width of the beam decreases the deflection of the sandwich beam with this difference that the natural frequency remains constant.
7. HSAPT shows higher deflection for sandwich beam with respect to Euler Bernoulli theory.

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