Latin American Journal of Solids and Structures
www.lajss.org

Effect of Surface Stress on the Buckling of Nonlocal Nanoplates Subject to Material Uncertainty

Abstract
At the nano scale, the effect of surface stress becomes prominent as well as the so-called small scale effect. Furthermore due to difficulties in making accurate measurements at nano-scale as well as due to due to molecular defects and manufacturing tolerances, there exists a certain degree of uncertainty in the determination of the material properties of nano structures This, in turn, introduces some degree of uncertainty in the computation of the mechanical response of the nano-scale components. In the present study a convex model is employed to take surface tension, small scale parameter and the elastic constants as uncertain-but-bounded quantities in the buckling analysis of rectangular nanoplates. The objective is to determine the lowest buckling load for a given level of uncertainty to obtain a conservative estimate by taking the worst-case variations of material properties. Moreover the sensitivity of the buckling load to material uncertainties is also investigated.

Keywords
Nanoplates, surface stress, buckling, material uncertainty, non-local theory, sensitivity analysis.

I. S. Radebe [a]
S. Adali [b, *]

[a] Department of Mechanical Engineering, Durban University of Technology, Durban, South Africa
E-mail: sfisor@dut.ac.za
[b] Mechanical Engineering, University of KwaZulu-Natal, Durban, South Africa
E-mail: adali@ukzn.ac.za

http://dx.doi.org/10.1590/1679-78251468

Received 19.07.2014
In Revised Form 30.09.2014
Accepted 03.11.2014
Available online 11.11.2014

1 INTRODUCTION

At nano scales, surface area to volume ratio increases to the extent that the surface stress effects can no longer be ignored as noted by Miller and Shenoy (2000) and Sun and Zhang (2003). This phenomenon has been observed in a number of studies and, in particular, the effect of surface tension on the properties of nano-sized structures has been noted in (Cuenot et al., 2004; Jing et al., 2006; Park and Klein, 2008; Stan et al., 2008; Eremeyev et al., 2009; Wang et al., 2010). The reason for the surface effects is the difference in the behavior of atoms depending on whether they are close to a free surface or within the bulk of the material. Atoms close to or at the surface lead to higher stiffness and mechanical strength (Murdoch, 2005). A review of the effect of surface stress on nanostructures is given by Wang et al (2011).

Presently a number of structures at the nano-scale is being studied and these include one-dimensional nanowires and nanobeams as well as the two-dimensional nanoplates and graphene. Recent work on the effect of surface energy on the mechanical behavior of nanowires include (Jiang and Yan, 2010; Hasheminejad and Gheshlaghi, 2010; Lee and Chang, 2011; Samaei et al., 2012). Vibrations of nanobeams with surface effects have been studied by Gheshlaghi and Hasheminejad (2011), Sharabiani and Yazdi (2013), Hosseini-Hashemi and Nazemnezhad (2013), Malekzadeh and Shojaee (2013).

High area to volume ratio of nanoplates makes them particularly susceptible to surface effects and the accuracy of solutions improves by including these effects in the governing equations. Theory of plates with surface effects has been developed in (Lu et al., 2006). The effect of surface stress on the stiffness of cantilever plates was studied by Lachut and Sader (2007). Wang and Wang (2011a) and Farajpour et al. (2013) studied the buckling of nonlocal nanoplates and included the effects of surface energy to assess its effect on the buckling load. Several studies on the vibrations and dynamics of nanoplates were conducted taking the surface effects into account in (Ansari and Sahmani, 2011; Wang and Wang, 2011b; Assadi, 2013; Narendar and Gopalakrishnan, 2012).

In the above studies only the average values of the material properties were used and the possibility of variations and/or inaccuracies in the data was not considered. Main drawback of the studies using deterministic material properties of the nano-sized structures is that the elastic constants and other material properties such as surface tension and the small-scale parameter often cannot be determined with a high degree of accuracy. The uncertainties in determining the values of these constants are due to a number of reasons which include processing difficulties, measurement inaccuracies, and defects and imperfections in the molecular structures, to name a few. For example experimental difficulties for making accurate measurements at the nano scale can lead to significant scatter in material data as noted by Kis and Zetti (2008) and Lee et al. (2008).

Results obtained by neglecting the possibility of uncertainty in material properties are, in general, not reliable in the sense that the load carrying capacity of the structure may be overestimated (Au et al., 2003). In the case of buckling, premature buckling may occur when these uncertainties affect the structure in a negative way. However the uncertainty in the data can be incorporated into a non-deterministic model to improve the reliability of results and to compute a conservative buckling load. In the present study this is done by convex modeling which requires that the uncertain quantities are bounded by an ellipsoid (Luo et al., 2009). Examples of convex modeling applied to various engineering problems with uncertain data can be found in (Sadek et al., 1993; Adali et al., 1995; Qiu et al., 2009; Kang et al., 2011; Luo et al., 2011; Radebe and Adali, 2013), where beams, plates and columns have been studied with respect to static and dynamic response, vibration and buckling.

Nanoplates are used in several nanotechnology applications and often subjected to in-plane loads which can lead to failure by buckling, especially considering their extremely small thickness (Asemi et al., 2014a, 2014b). Buckling behavior and sensitivity of nonlocal orthotropic nanoplates with material uncertainty have been investigated by Radebe and Adali (2014) neglecting the surface effect. Present study extends these results to take into account the effect of surface stress in the buckling of isotropic nanoplates. The material parameters taken as uncertain are residual surface stress, surface elastic modulus, the small scale parameter of the nonlocal theory and Young's modulus. To investigate the sensitivity of the buckling load to uncertain material properties, relative sensitivity indices are defined (Cacuci, 2003; Conceição António and Hoffbauer, 2013). The effect of uncertainty on the buckling load is studied in the numerical examples and the sensitivity to uncertainty is studied by means of contour plots.

2 NONLOCAL NANOPLATE WITH SURFACE EFFECTS

We consider a rectangular nanoplate subject to biaxial buckling loads N_x and N_y acting in the x and y directions, respectively. The dimensions of the plate are specified as a in the x-direction and b in the y-direction with the plate thickness given by h. The differential equation governing the buckling of the nanoplate based on nonlocal elastic theory and including the effect of surface energy is given in (Wang and Wang, 2011a) as

$$\left(\tilde{D}+\frac{1}{2}h^2\tilde{E}^s\right)\nabla^4 w-\frac{1}{2}\tilde{\mu}^2h^2\tilde{E}^s\nabla^6 w-(1-\tilde{\mu}^2\nabla^2)\left(2\tilde{\tau}_0\nabla^2 w+N_x\frac{\partial^2 w}{\partial^2 x}+N_y\frac{\partial^2 w}{\partial^2 y}\right)=0 \tag{1}$$

where $w(x,y)$ is the deflection of the plate, $\nabla^2=\frac{\partial^2 v}{\partial^2 x}+\frac{\partial^2 v}{\partial^2 y}$, $\tilde{D}=\frac{\tilde{E}h^3}{12(1-\nu^2)}$, $\tilde{\mu}$ is the small-scale parameter of the nonlocal theory and ν is the Poisson's ratio. Uncertain quantities are denoted by the tilde symbol. The uncertain material constants are $\tilde{E}$ (Young's modulus), $\tilde{E}^s$ (surface elastic modulus) and $\tilde{\tau}_0$ (residual surface stress). For a simply supported plate, the solution is given by

$$w(x,y)=\sum_m\sum_n c_{mn}\sin(m\pi x/a)\sin(n\pi y/b) \tag{2}$$

The buckling load can be obtained by substituting Eq. (2) into Eq. (1). This computation gives (Wang and Wang, 2011a)

$$N_{\text{cr}}=\min_{m,n}\frac{\eta^4(\tilde{D}+0.5h^2\tilde{E}^s+2\tilde{\tau}\tilde{\mu}^2)+0.5h^2\eta^6\tilde{E}^s\tilde{\mu}^2+2\eta^2\tilde{\tau}_0}{(\xi^2+R\zeta^2)\left(1+\eta^2\tilde{\mu}^2\right)} \tag{3}$$

where $\xi=m\pi/a$, $\zeta=n\pi/b$, $\eta^2=\xi^2+\zeta^2$, $R=N_y/N_x$ and $m,n=1,2,3\ldots$ By defining the constants g_1 and g_2 given by

$$g_1 = \frac{h^3}{12(1-\nu^2)}, \qquad g_2 = 0.5\,h^2 \tag{4}$$

the buckling load given by Eq. (3) can be expressed as

$$N_{\mathrm{cr}} = \min_{m,n} \frac{\eta^4(g_1\tilde{E} + g_2\tilde{E}^s + 2\tilde{\tau}\tilde{\mu}^2) + g_2\eta^6\tilde{E}^s\tilde{\mu}^2 + 2\eta^2\tilde{\tau}_0}{(\xi^2 + R\zeta^2)\left(1+\eta^2\tilde{\mu}^2\right)} \tag{5}$$

Introducing the uncertainty parameters ε_i, the uncertain material constants can be expressed as

$$\tilde{E} = E_0(1+\varepsilon_1), \qquad \tilde{E}^s = E_0^s(1+\varepsilon_2), \qquad \tilde{\tau} = \tau_0(1+\varepsilon_3), \qquad \tilde{\mu} = \mu_0(1+\varepsilon_4) \tag{6}$$

where the subscript "0" denotes the nominal quantities and $|\varepsilon_i| << 1$ is the amount of uncertainty for the corresponding material property and can be positive or negative. The parameters ε_i are unknown and have to be determined to obtain the so-called "worst-case" buckling load which corresponds to the lowest buckling load for a given level of uncertainty. Substituting Eq. (6) into Eq. (5), neglecting the terms with ε_i^2 and higher order, and keeping only the terms linear in ε_i, we obtain

$$N_{\mathrm{cr}}(\varepsilon_i) = \min_{m,n} \frac{A_0 + g_1\,\eta^4 E_0\,\varepsilon_1 + g_2\,\eta^4 E_0^s\gamma\,\varepsilon_2 + 2\eta^2\tau_0\,\gamma\,\varepsilon_3 + 2\eta^4\mu_0^2(2\tau_0 + g_2\,\eta^2 E_0^s)\,\varepsilon_4}{(\xi^2 + R\zeta^2)\gamma^2\left(1+2\eta^2\mu_0^2\gamma^{-1}\varepsilon_4\right)} \tag{7}$$

where

$$A_0 = \eta^4(g_1E_0 + g_2E_0^s) + 2\eta^4\tau_0\mu_0^2 + g_2\eta^6E_0^s\mu_0^2 + 2\eta^2\tau_0, \qquad \gamma = 1 + \eta^2\mu_0^2 \tag{8}$$

The expression (7) for $N_{\mathrm{cr}}(\varepsilon_i)$ can be approximated as

$$N_{\mathrm{cr}}(\varepsilon_i) \cong \min_{m,n}\left(a_{0mn} + \sum_{i=1}^{4} a_{imn}\varepsilon_i \right) \tag{9}$$

by linearizing it with respect to the uncertainty parameters ε_i. This is achieved by using the relation $(1 \pm \varepsilon)^c \cong (1 \mp c\varepsilon) + O(\varepsilon^2)$ where the superscript c can take positive or negative values and $|\varepsilon| << 1$. The values of a_{imn} appearing in Eq. (9) are given in the Appendix.

3 CONVEX MODELING

In the present section a convex model is implemented to investigate the effect of uncertainties on the buckling load. The objective is to determine the uncertainty parameters ε_i such that the most conservative buckling load is obtained in the presence of material uncertainties. Implementing the convex modeling , the parameters ε_i are bounded such that they satisfy the inequality

$$\sum_{i=1}^{4} \varepsilon_i^2 \leq \beta^2 \tag{10}$$

where β is the radius of a 4-dimensional ellipsoid. As such β is a measure of the level of uncertainty and satisfies the inequality $\beta < 1$ since $|\varepsilon| << 1$. It is known that the buckling load takes its extreme values on the boundary of the ellipsoid defined by Eq. (10) (see Sadek et al., 1993; Adali et al., 1995). Thus the inequality (10) can be replaced by the equality

$$\sum_{i=1}^{n} \varepsilon_i^2 = \beta^2 \tag{11}$$

to compute the "worst-case" solution. The expression (9) for N_{cr} is to be minimized subject to the constraint (11) to compute the constants ε_i and to obtain the lowest buckling load. For the constrained optimization problem, the following Lagrangian, denoted by L, is introduced

$$L(a_i, \varepsilon_i) = a_{0mn} + \sum_{i=1}^{4} a_{imn} \varepsilon_i + \lambda \left(\sum_{i=1}^{4} \varepsilon_i^2 - \beta^2 \right) \tag{12}$$

By setting $\partial L(a_{imn}, \varepsilon_i) / \partial \varepsilon_i = 0$ and using Eq. (11), the parameters ε_i and the Lagrange multiplier λ are computed as

$$\varepsilon_i = -\frac{a_{imn}}{2\lambda}, \qquad \lambda = \pm \frac{1}{2\beta} \left(\sum_{i=1}^{4} a_{imn}^2 \right)^{1/2} \tag{13}$$

Thus

$$\varepsilon_i = \mp \beta a_{imn} \left(\sum_{i=1}^{4} a_{imn}^2 \right)^{-1/2} \qquad \text{for} \qquad i = 1,2,3,4 \tag{14}$$

where the plus and minus signs correspond to the lowest and highest buckling loads.

4 SENSITIVITY ANALYSIS

The sensitivity of the buckling load to uncertain parameters can be investigated by defining relative sensitivity indices $S_K(\varepsilon_i)$ given by

$$S_K(\varepsilon_i) = \left|\frac{\partial N_{cr}(\beta)}{\partial \varepsilon_i}\right| \frac{|\varepsilon_i|}{N_{cr}(0)} \tag{15}$$

which is normalized with respect to the deterministic buckling load $N_{cr}(0)$. In Eq. (15), the sensitivities of the buckling load to $\tilde{E}$, $\tilde{E}^s$, $\tilde{\tau}$ and $\tilde{\mu}$ with respect to uncertainty parameters ε_i, $i=1,2,3,4$ are denoted by $S_E(\varepsilon_1)$, $S_{E^s}(\varepsilon_2)$, $S_\tau(\varepsilon_3)$ and $S_\mu(\varepsilon_4)$, respectively, so that the subscript K stands for the respective material property. Equation (15) indicates that the buckling pressure has zero sensitivity for $\varepsilon_i = 0$ corresponding to the deterministic case as expected. The sensitivities $S_K(\varepsilon_i)$ can be computed from Eqs. (9) and (15) as

$$S_K(\varepsilon_i) = \frac{|a_{imn}\,\varepsilon_i|}{a_{0mn}} \tag{16}$$

noting that $N_{cr}(0) \equiv a_{0mn}$.

5 NUMERICAL RESULTS

The effect of uncertainties in the material properties on the buckling load and the sensitivity of the buckling load to the level of uncertainty are numerically studied in the present section. The results are given for a square silver nanoplate of thickness $h = 5$ nm. The nominal values of the elastic constants are taken as $E_0 = 76$ GPa, $\nu = 0.3$, $E_0^s = 1.22$ N/m and $\tau_0 = 0.89$ N/m which are the values used in (Wang and Wang 2011a). The buckling load is normalized with respect to the buckling load N_L of a plate without surface and nonlocal effects and having constants corresponding to the nominal values of the material properties. The buckling load N_L can be obtained from Eq. (7) by setting $E_0^s = \tau_0 = \mu_0 = \varepsilon_i = 0$. Thus the normalized buckling load is given by $N_R = N_{cr}(\varepsilon_i)/N_L$.

Figure 1 shows the curves of N_R plotted against the length a for various uncertainty levels with $\mu_0 = 2$ nm under the biaxial loading $N_x = N_y$ ($R=1$). It is noted that the buckling load curve for the deterministic case ($\beta = 0.0$) corresponds to the result given in Figure 1 of Wang and Wang (2011a). The corresponding results for the uniaxial loading with $N_y = 0$ ($R=0$) are given in Figure 2. Both figures show that the buckling load decreases as the uncertainty in the material properties increases.

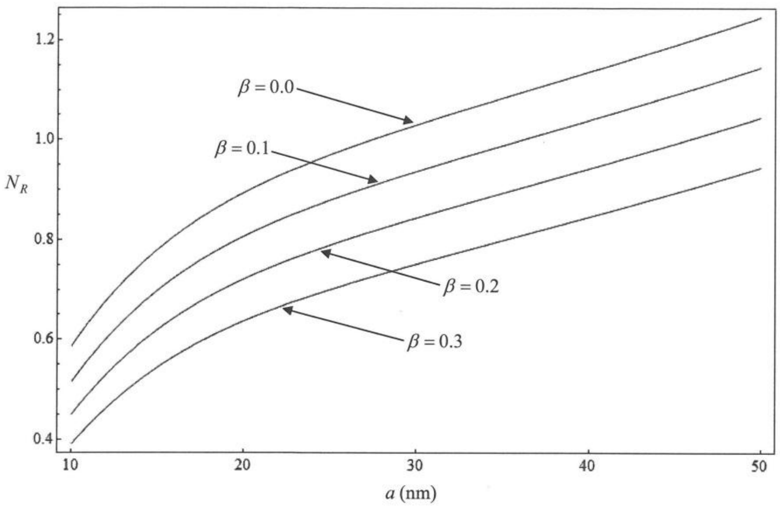

Figure 1: Curves of N_R plotted against a for various uncertainty levels with $b = a$, $\mu_0 = 2$ nm and $N_y / N_x = 1$.

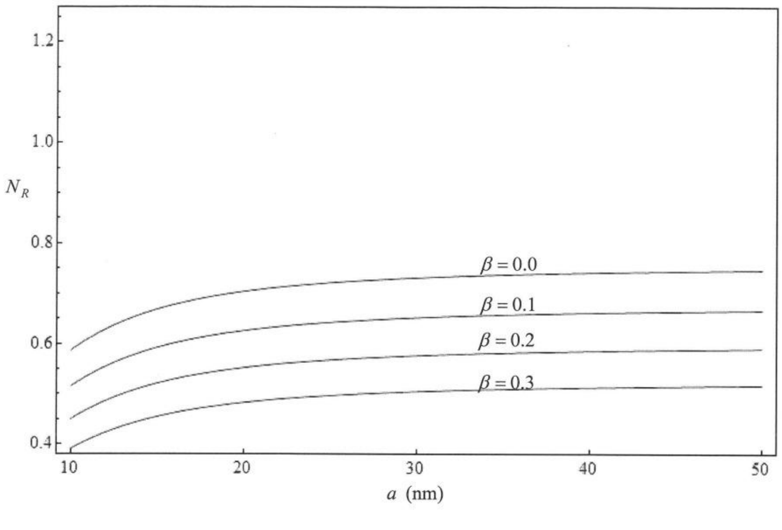

Figure 2: Curves of N_R plotted against a for various uncertainty levels with $b = a$, $\mu_0 = 2$ nm and $N_y = 0$.

The sensitivity results are given in Figure 3 with respect to $\tilde{E}$, $\tilde{E}^s$, and in Figure 4 with respect to $\tilde{\tau}$ and $\tilde{\mu}$ which show the contour plots of the sensitivity indices given by Eq. (16) plotted against the level of uncertainty and the length a for a square nanoplate with $N_y / N_x = 1$. In all cases the sensitivity of the buckling load increases with increasing uncertainty. Figure 3a indicates that the buckling load is most sensitive to changes in $\tilde{E}$ and least sensitive to changes in the surface modulus $\tilde{E}^s$ (Figure 3b). The second most sensitivity is observed towards the uncertainty in the small-scale parameter $\tilde{\mu}$ (Figure 4b). Sensitivities to $\tilde{\mu}$ and $\tilde{E}^s$ (Figs. 3b and 4b) increase as the size of the nanoplate becomes smaller, i.e., as $a \to 0$. On the other hand the sensitivity towards the residual surface stress $\tilde{\tau}_0$ increases as the nanoplate becomes larger (Fig. 4a).

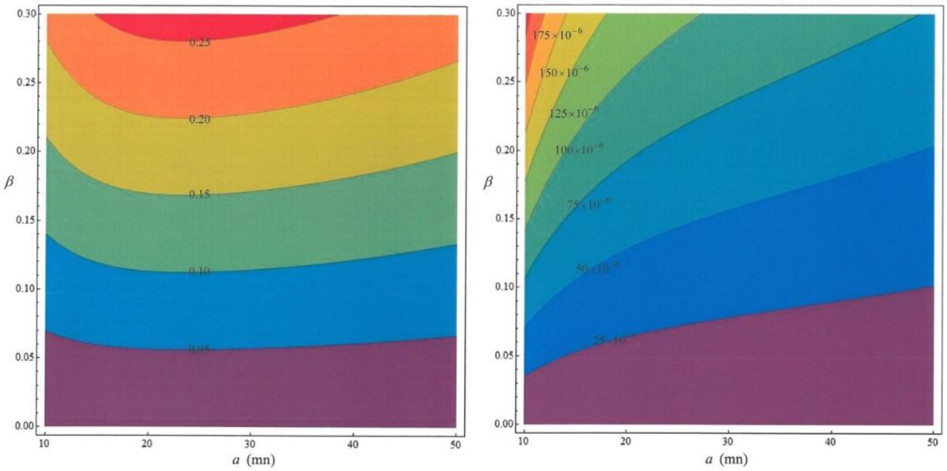

Figure 3: Contour plots of sensitivities plotted against uncertainty level β and a with $b = a$, $\mu_0 = 2$ nm and $N_y / N_x = 1$, a) $S_E(\varepsilon_1)$, b) $S_{E^s}(\varepsilon_2)$.

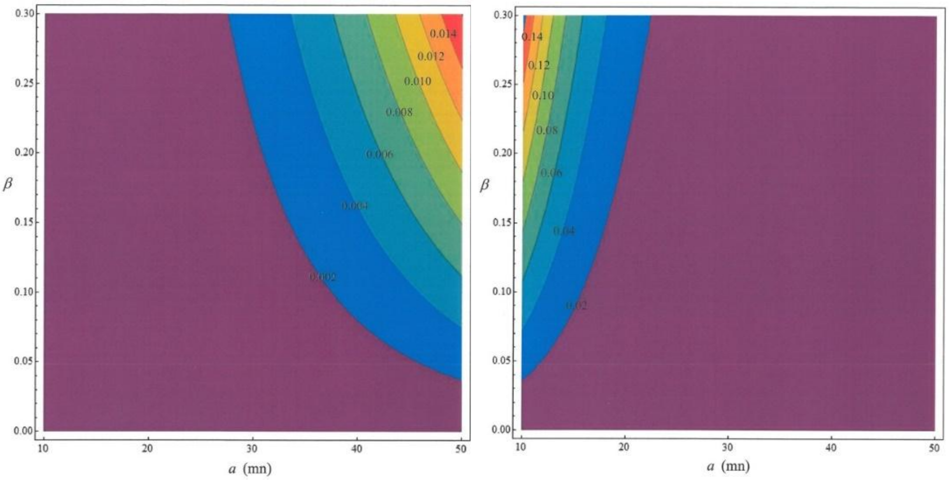

Figure 4: Contour plots of sensitivities plotted against uncertainty level β and a with $b = a$, $\mu_0 = 2$ nm and $N_y / N_x = 1$, a) $S_\tau(\varepsilon_3)$, b) $S_\mu(\varepsilon_4)$.

6 CONCLUSIONS

Material properties at the nanoscale are usually known with a certain level of tolerance. The present study is directed to determining the buckling loads of nano-scale plates with material uncertainties and including surface stress the effect of which cannot be neglected for nanoplates. The uncertain parameters are the Young's modulus, surface elastic modulus, residual surface stress and small scale parameter of the nonlocal theory. The effect of uncertainty on the buckling load is studied and the sensitivity of the buckling load to uncertain parameters is investigated. The uncertainty is taken into account by convex modeling which determines the worst-case combination of material properties to determine the lowest buckling load for a given level of uncertainty.

In the present case convex modeling leads to a four-dimensional ellipsoid bounding the uncertain parameters and the method of Lagrange multipliers is employed to compute these parameters. Moreover sensitivity indices are defined to investigate the relative sensitivity of the buckling load to uncertainties in the elastic constants. The numerical results show the effect of increasing uncertainty on the buckling load for biaxial and uniaxial buckling loads (Figures 1 and 2). The sensitivity studies indicate that the buckling load is most sensitive to uncertainty in Young's modulus and the size of the nanoplate affects various sensitivities in different ways. It is observed that the sensitivities to surface elastic modulus and small-scale parameter decrease (Figures 3b and 4b) and the sensitivity to residual surface stress increases (Figure 4a) as the nanoplate becomes larger. Sensitivity to Young's modulus is mostly influenced by the level of uncertainty (Figures 3a).

Acknowledgements

The research reported in this paper was supported by research grants from the University of KwaZulu-Natal (UKZN) and from National Research Foundation (NRF) of South Africa. The authors gratefully acknowledge the supports provided by UKZN and NRF.

References

Adali, S., Bruch, Jr., J. C., Sadek, I. S., Sloss, J. M. (1995). Transient vibrations of cross-ply plates subject to uncertain excitations. Applied Mathematical Modelling 19: 56–63.

Ansari, R., Sahmani, S. (2011). Surface stress effects on the free vibration behavior of nanoplates. International Journal of Engineering Science 49: 1204–1215.

Assadi, A. (2013). Size dependent forced vibration of nanoplates with consideration of surface effects. Applied Mathematical Modelling 37: 3575–3588.

Asemi, A. S., Farajpour, A., Borghei, M., Hassani, A. H. (2014a). Thermal effects on the stability of circular graphene sheets via nonlocal continuum mechanics. Latin American Journal of Solids and Structures 11: 704-724.

Asemi, A. S., Mohammadi, M., Farajpour, A. (2014b). A study on the nonlinear stability of orthotropic single-layered graphene sheet based on nonlocal elasticity theory. Latin American Journal of Solids and Structures 11: 1541-1564.

Au, F. T. K., Cheng, Y. S., Tham, L. G., Zeng, G. W. (2003). Robust design of structures using convex models. Computers and Structures 81: 2611–2619.

Cacuci, D. G. (2003). Sensitivity and Uncertainty Analysis, Vol. 1: Theory. Boca Raton, FL: Chapman & Hall/CTC Press.

Conceição António, C. A., Hoffbauer, L. N. (2013). Uncertainty assessment approach for composite structures based on global sensitivity indices. Composite Structures 99: 202–212.

Cuenot, S., Frétigny, C., Demoustier-Champagne, S., Nysten, B. (2004). Surface tension effect on the mechanical properties of nano materials measured by atomic force microscopy. Physical Reviews B 69: 165410–165413.

Eremeyev, V. A., Altenbach, H., Morozov, N. F. (2009). The influence of surface tension on the effective stiffness of nanosize plates. Doklady Physics 54: 98–100.

Farajpour, A., Dehghany, M., Shahidi, A.R. (2013). Surface and nonlocal effects on the axisymmetric buckling of circular graphene sheets in thermal environment. Composites Part B: Engineering 50: 333–343.

Gheshlaghi, B., Hasheminejad, S. M. (2011). Surface effects on nonlinear free vibration of nanobeams. Composites Part B: Engineering 42: 934–937.

Hasheminejad, S. M., Gheshlaghi, B. (2010). Dissipative surface stress effects on free vibrations of nanowires. Applied Physics Letters 97: 253103.

Hosseini-Hashemi, S., Nazemnezhad, R. (2013). An analytical study on the nonlinear free vibration of functionally graded nanobeams incorporating surface effects. Composites Part B: Engineering 52: 199–206.

Jiang, L. Y., Yan, Z. (2010). Timoshenko beam model for static bending of nanowires with surface effects. Physica E 42: 2274–2279.

Jing, G. Y., Duan, H. L., Sun, X. M., Zhang, Z. S., Xu, J., Li, Y. D., Wang, J. X., Yu, D. P. (2006). Surface effects on elastic properties of silver nanowires: Contact atomic-force microscopy. Physical Reviews B 73: Art. 235409.

Kang, Z., Luo, Y., Li, A. (2011). On non-probabilistic reliability-based design optimization of structures with uncertain-but-bounded parameters. Structural Safety 33: 196–205.

Kis, A., Zetti, A. (2008). Nanomechanics of carbon nanotubes. Philosophical Transactions of Royal Society A 366: 1591–1611.

Lachut, M. J., Sader, J. E. (2007). Effect of surface stress on the stiffness of cantilever plates. Physics Review Letters 99: 206102.

Lee, C., Wei, X., Kysar, J.W., Hone, J. (2008). Measurement of the elastic properties and intrinsic strength of monolayer graphene. Science 321: 385–388.

Lee, H. L., Chang, W. J. (2011). Surface effects on axial buckling of nonuniform nanowires using non-local elasticity theory. Micro and Nano Letters 6: 19–21.

Lu, P., He, L. H., Lee, H. P., Lu, C. (2006). Thin plate theory including surface effects. International Journal of Solids and Structures 43: 4631–4647.

Luo, Y., Kang, Z., Luo, Z., Li, A. (2009). Continuum topology optimization with non-probabilistic reliability constraints based on multi-ellipsoid convex model. Structural and Multidisciplinary Optimization 39: 297–310.

Luo, Y., Li, A., Kang, Z. (2011). Reliability-based design optimization of adhesive bonded steel-concrete composite beams with probabilistic and non-probabilistic uncertainties. Engineering Structures 33: 2110–2119.

Malekzadeh, P., Shojaee, M. (2013). Surface and nonlocal effects on the nonlinear free vibration of non-uniform nanobeams. Composites Part B: Engineering 52: 84–92.

Miller, R. E., Shenoy, V. B. (2000). Size-dependent elastic properties of nanosized structural elements. Nanotechnology 11: 139–147.

Murdoch, A. I. (2005). Some fundamental aspects of surface modelling. Journal of Elasticity 80: 33–52.

Narendar, S., Gopalakrishnan, S. (2012). Study of terahertz wave propagation properties in nanoplates with surface and small scale effects. International Journal of Mechanical Science 64: 221–231.

Park, H. S., Klein, P. A., (2008). Surface stress effects on the resonant properties of metal nanowires: The importance of finite deformation kinematics and the impact of the residual surface stress. Journal of Mechanics and Physics of Solids 56: 3144–3166.

Qiu, Z., Ma, L., Wang, X. (2009). Unified form for static displacement, dynamic response and natural frequency analysis based on convex models, Applied Mathematical Modelling 33: 3836–3847.

Radebe, I. S., Adali, S. (2013). Minimum weight design of beams against failure under uncertain loading by convex analysis. Journal of Mechanical Science and Technology 27: 2071–2078.

Radebe, I. S., Adali, S. (2014). Buckling and sensitivity analysis of nonlocal orthotropic nanoplates with uncertain material properties. Composites Part B: Engineering 56: 840–846.

Sadek, I. S., Sloss, J. M., Adali, S., Bruch, Jr., J. C. (1993). Nonprobabilistic modelling of dynamically loaded beams under uncertain excitations. Mathematical and Computer Modelling 18: 59–67.

Samaei, A. T., Bakhtiari, M., Wang, G.-F. (2012). Timoshenko beam model for buckling of piezoelectric nanowires with surface effects. Nanoscale Research Letters 7: 201 (6 pages).

Sharabiani, P. A., Yazdi, M. R. H. (2013). Nonlinear free vibrations of functionally graded nanobeams with surface effects. Composites Part B: Engineering 45: 581–586.

Stan, G., Krylyuk, S., Davydov, A. V., Vaudin, M., Bendersky, L. A., Cook, R. F. (2008). Surface effects on the elastic modulus of Te nanowires. Applied Physics Letters 92: 241908.

Sun, C. T., Zhang, H. (2003). Size-dependent elastic moduli of platelike nanomaterials. Journal of Applied Physics 93: 1212–1218.

Wang, J., Huang, Z., Duan, H., Yu, S., Wang, X. G., Zhang, W., Wang, T. (2011). Surface stress effect in mechanics of nano structured materials. Acta Mechanica Solida Sinica 24: 52–82.

Wang, K. F., Wang, B. L. (2011a). Combining effects of surface energy and non-local elasticity on the buckling of nanoplates. Micro and Nano Letters 6: 941–943.

Wang, K. F., Wang, B. L. (2011b). Vibration of nanoscale plates with surface energy via nonlocal elasticity. Physica E 44: 448–453.

Wang, Z. Q., Zhao, Y. P., Huang, Z. P. (2010). The effects of surface tension on the elastic properties of nano structures. International Journal of Engineering Science 48: 140–150.