www.lajss.org
Latin American Journal of Solids and Structures 5 (2008) 75–95

Parametric uncertainty analysis considering metrological aspects in the structural simulation in viscoelastic materials

Antonio de Assis Brito Neto* and Edison da Rosa

Departamento de Engenharia Mecnica, Universidade Federal de Santa Catarina,
Santa Catarina, SC – Brasil

Abstract

This paper proposes to discuss the necessity of including metrological aspects in the parametric uncertainty analysis in the structural simulation process in viscoelastic materials. The research focus is the identification, quantification and propagation of the measurement uncertainty (involved in the problem modeling), through the computational model of structural simulation. There are many experimental procedures supporting the characterization of the input parameters in a structural model such as the definition of the structural geometry, levels of applied loads, material properties and ambient conditions. Consequently, the usage of metrological aspects becomes necessary in order to guarantee the equivalence of results among different laboratories and also the evaluation of the measurement or simulation result with its specifications. The proposed methodology for analyzing the uncertainties present in the simulation of structural problems in viscoelastic materials is composed by two modules. The first module consists on identifying and quantifying the sources of uncertainty present in the process of characterizing a viscoelastic material through the creep test. In the second module the influence of material properties and other uncertainties in the simulated behavior of the structural component is investigated. The Monte Carlo simulation method is used for this purpose. The Monte Carlo method of simulation has certain limitations in its application to viscoelastic problems, due to the need of repeated simulations, the time of computational analysis raises. This paper presents alternatives for using SMC in viscoelastic problems through models supplied by the correspondence principle and the response surface method.

Keywords: metrology, uncertainty analysis, structural simulation, viscoelasticity.

1 Introduction

This work proposes to discuss the inclusion of metrological aspects in the structural simulation process, having as a case study the analysis of components in viscoelastic materials. The work focus is the uncertainty analysis, whose main objective is to present the methodology used to identify and quantify the propagation of the influence of measurement uncertainty (presented

*Corresp. author email: britoneto@yahoo.com.br Received 17 Dec 2007; In revised form 19 Feb 2008

in the process of input parameters determination) in the results supplied by computational simulation models of structural problems in viscoelastic materials (figure 1). This methodology will help answer important questions to guarantee the reliability and efficiency of structural projects, such as:

1. What is the influence of certain levels of uncertainty of an input parameter in the output of the simulation model?

2. What level of uncertainty can be allowed in the measurements processes, used in the characterization of the input parameters of a simulation model, in order to obtain a determined uncertainty level in the model answer?

3. Which main source of uncertainty should be minimized in order to obtain an acceptable level of uncertainty in the model answer inside of the structural component requirements?

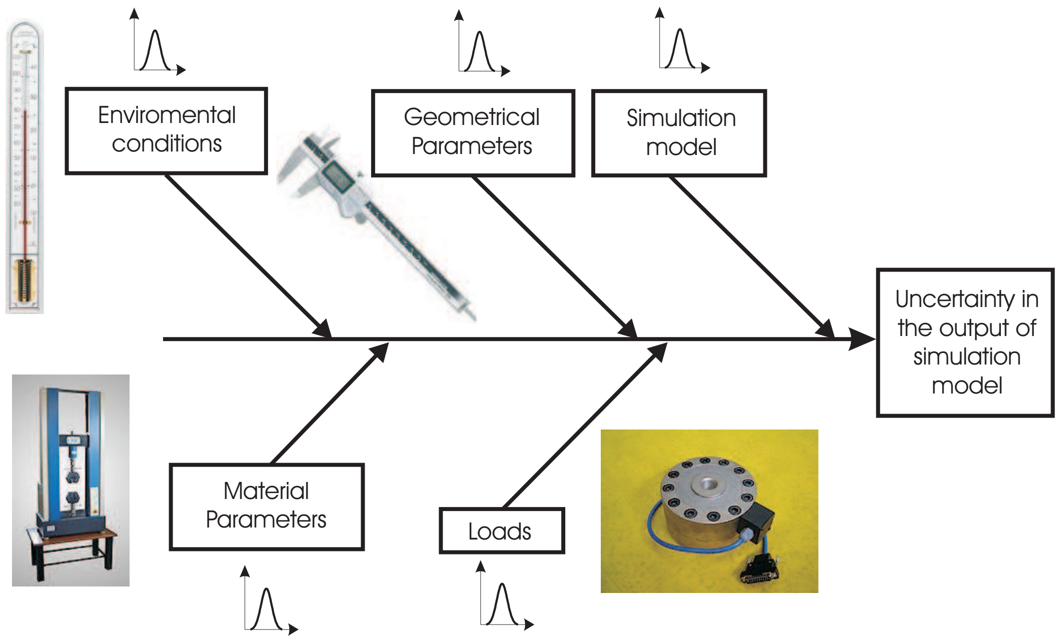

Figure 1: Propagation of measurement uncertainties through the structural simulation model.

The methodology proposed in the solution problem is based on four modules:

I. Material modeling - the study and implementation of mathematical model algorithms for viscoelastic materials in the time domain, using classic and fractionary rheological models.

II. Material characterization - the study and accomplishment of experimental creep tests for viscoelastic material characterization and the determination of material parameter uncertainty.

III. Structural simulation - the study of finite element method for viscoelastic problems and correspondence principle method.

IV. Analysis and propagation of uncertainties - the study of uncertainty propagation techniques and application of metrological concepts for identification, quantification and propagation of measurement uncertainties involved in the structural modeling in viscoelastic material.

2 Mathematical material models for linearly viscoelastic response

Materials which exhibit both viscous fluid and elastic solid characteristics are called viscoelastic materials. The description of the response of materials which exhibit combined viscous and elastic properties is based upon an analogy with the response of certain mechanical elements. This involves the construction of viscoelastic models by the combining mechanical elements which represent viscous and elastic properties.

The constitutive law for a linearly viscoelastic material can be represented through hereditary integrals, where the uni-axial relationship between stress and strain can be written as:

$$\varepsilon\left(t\right) = J\left(0\right)\sigma\left(t\right) + \int\limits_{0}^{t} J\left(t-\tau\right)\frac{d\sigma}{dt}d\tau \tag{1}$$

where ε- strain, σ - stress, $J(t)$ is defined as creep compliance and $J(0)$ is the initial value of J(t) [6].

For classic rheological models (is shown in figure 2 the generalized Maxwell model), the creep compliance $J(t)$, is given by a sum of exponential functions well-known as Prony series [10]

$$J\left(t\right) = J_{\mathrm{inf}} - \sum_{i=1}^{n} J_i e^{-t/\tau_i} \tag{2}$$

where τ_i times of relaxation, J_i material parameters and J_{inf} creep compliance for $t = \infty$.

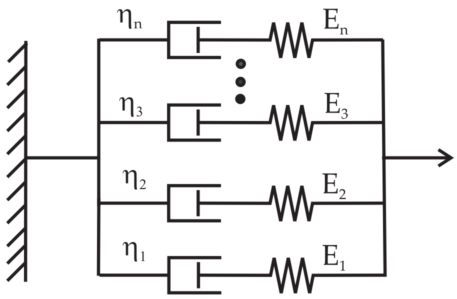

Figure 2: Generalized Maxwell Model.

With the development of fractional calculus, Koeller [13] developed a new rheological element, the "spring-pot". The constitutive equation of this element possesses derivatives of fractional order, which mixed the behavior between an elastic and viscous material

$$\sigma = p\frac{\partial^\alpha \varepsilon}{\partial t^\alpha} \tag{3}$$

where p is a proportionality factor and α is the order of derivative which is commonly taken to range between 0 and 1. If α is 0, equation (3) describes the behavior of a spring where p specifies the springs' stiffness. For $\alpha = 1$, (3) defines the constitutive equation of a dashpot, in which p defines the viscosity. Thus, the fractional constitutive equation (3) 'interpolates' between the material behavior of a spring and that of a dashpot. The spring-pot element essentially substitutes the dashpot in the rheological classic models. The figure 3 shows the Fractional Zener model.

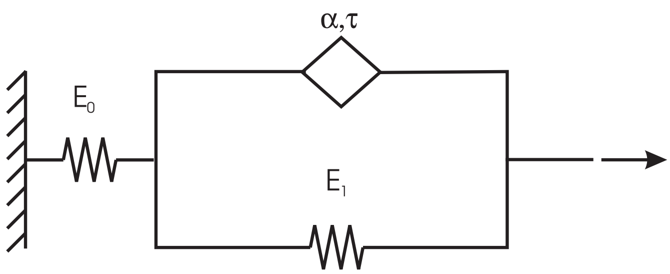

Figure 3: Fractional Zener model.

In agreement with Bagley [3], the viscoelastic behavior of a great amount of polymeric materials can be represented with the use of a fractional model with only four parameters. The fractional model used in this work is the Fractional Zener model [14], where the creep compliance is given by the equation

$$J(t) = \frac{1}{E_0}\left\{1 + \frac{E_0}{E_1}\left(1 - \mathcal{E}_\alpha\left[-\left(\frac{t}{\tau}\right)^\alpha\right]\right)\right\} \tag{4}$$

where E_0, E_1 and τ material parameters, as well as the fractional order derivative α. The nucleus of this function is given by the Mittag-Leffler function, $E_\alpha(*)$, which is defined by an infinite series [8]

$$E_\alpha(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k + 1)} \tag{5}$$

where $\Gamma(*)$ represents the Gamma function. The references [7, 11] present algorithms for the implementation of this function.

The finite element method [15] will be used in the simulation of structural problems through commercial software ANSYS [2].

3 The classic and numerical correspondence principle

The classic correspondence principle, CP, states that if a solution to a linear elasticity problem is known, the solution to the corresponding problem for a linearly viscoelastic material can be obtained by replacing each quantity which can depend on time by its Laplace transform multiplied by the transform variable (p or s), and then by transforming back to the time domain. There is the restriction that the interface between boundaries under prescribed load and boundaries under prescribed displacement may not change with time, although the loads and displacement can be time dependent [9]. The case study, to be presented, will illustrate the application of the correspondence principle to the problem of a viscoelastic cantilever beam with constant load in its end.

An example of the application of correspondence principle is in the viscoelastic solution of a beam under uniform distributed load (figure 4). The elastic deflection this beam is given by the equation

$$w(x) = \frac{\bar{p}}{24IE}\left(x^4 - 2Lx^3 + L^3x\right) \tag{6}$$

where $\bar{p}$ is the uniform distribute load, I is the area moment of inertia ($bh^3/12$) and E is the Young modulus. The uniformly distributed load is applied as a creep load at $t = 0$ (i.e., the load is applied suddenly at $t = 0$ and then held constant). Due the simplicity of the applied load, the inverse Laplace transformation is facilitate. The viscoelastic solution for the quasi-static case under creep loading applied at $t = 0$ has the following result:

$$w(x,t) = \frac{\bar{p}}{24I}\left(x^4 - 2Lx^3 + L^3x\right) J(t)\,. \tag{7}$$

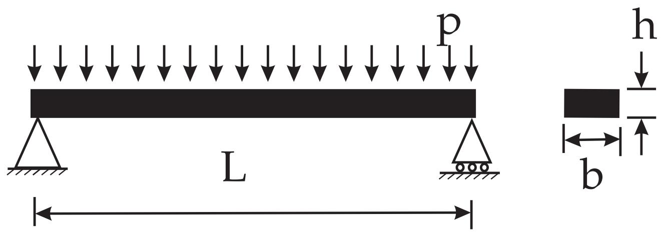

Figure 4: A simply supported beam with uniform distributed load.

This form of the correspondence principle is of a sweeping generality, but it does have its limitations. There are cases in which a Laplace transformation in not possible [10] and cases where the elastic solution is not available. The use of the correspondence principle on the generalized Hooke's law will be defined as numerical correspondence principle, NCP. In this method the stress and/or strain will be obtained through a numeric method (i.e. the finite element method), considering the material with purely elastic behavior. The main advantages of numeric correspondence principle is the possibility of solution problems by the CP where analytic elastic solution is not available.

4 PVC viscoelastic characterization

In this work the tensile creep test was used to characterize the behavior of polyvinyl chloride (PVC), being the objective of characterization test to supply data to determine the material parameters.

4.1 The creep test

The creep test consists of measuring the time dependent strain resulting from the application of a steady uniaxial stress. The norm ASTM D 2990-01 [1] establishes the requirements and the necessary procedure for the accomplishment of the creep test in plastics.

In this work, the experimental apparatus used for accomplishment of the creep test is presented in the figure 5. The experimental apparatus consists basically of a system to apply the load to the test body, a stove for temperature control and a measurement system for accompaniment of test body deformation. The test body deformation is measured through the use of EXCEL strain gage and HBM measuring amplifier system. The figure 6 shows an instrumented specimen.

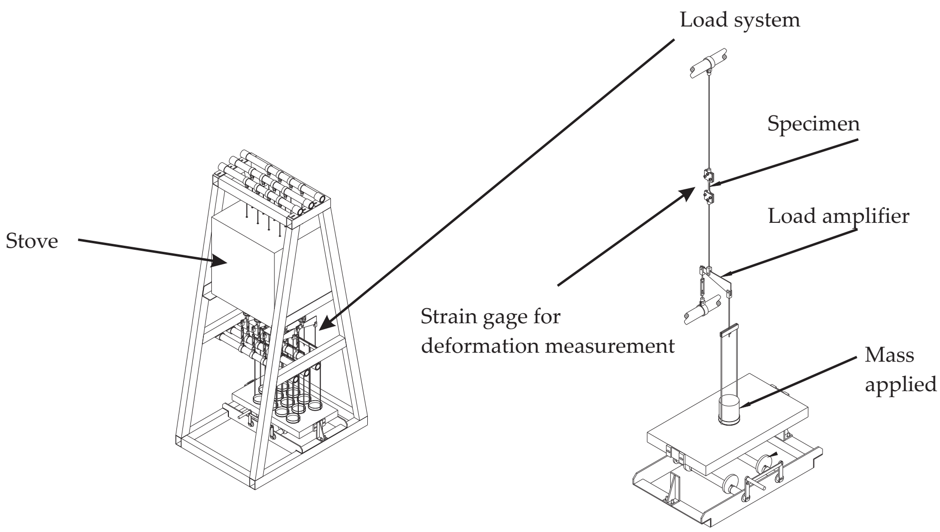

Figure 5: Experimental apparatus used in the creep test.

Figure 6: Instrumented specimens.

4.2 The adjust of material parameters

With base in the deformation of specimen, measured through the creep test, determination of the creep compliance parameters is obtained through an adjustment of experimental points to the mathematical model of material behavior. A process of nonlinear optimization is used. The optimization process consists in the minimization of least-square function

$$F(x,t) = \min_{x} \frac{1}{2} \sum_{i=1}^{n} \left(J\left(x,t_i\right) - J_{m_i}\right)^2 \tag{8}$$

where $J(t)$ given values by mathematical model of creep compliance and $J_m(t)$ the value of creep compliance obtained experimentally, i.e., $J_m(t) = \varepsilon(t)/\sigma_0$, where $\varepsilon(t)$ is the measured deformation in the specimen and σ_0 is the magnitude tension applied in the specimen. The illustration (7) presents the results of creep test bodies and fittings to the classic and fractional rheological models given by the equations (2) and (3).

5 Uncertainty analysis

In the structural analysis, it becomes necessary to determine the related parameters with respect to the geometry component, the applied load, the material properties, and the contour conditions of the problem through of measurement processes. The main uncertainty sources associated with the measurement processes are: instrument resolution; inherited uncertainty; environmental factors like variations and thermal gradients; vibrations; influence operator and measurement procedure, etc..

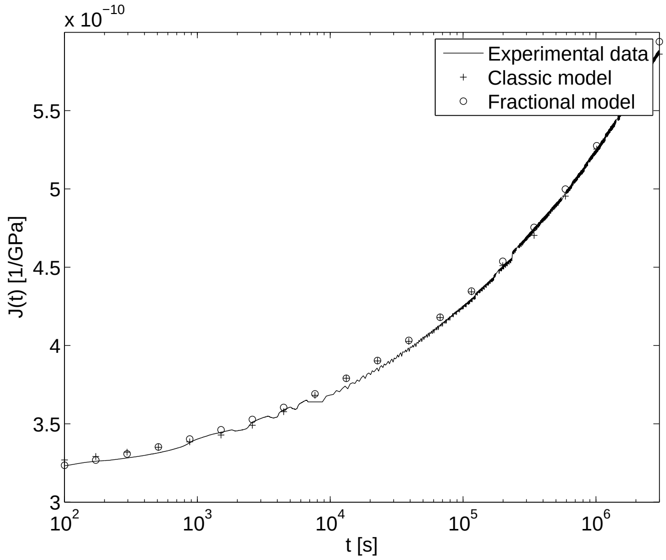

Figure 7: Results of curve-fitting of experimental data to classic and fractionary material models.

5.1 The method of Monte Carlo simulation

A measurement/simulation model is expressed by a functional relationship f:

$$y = f\left(x_1, x_2, \ldots, x_n\right) \tag{9}$$

where y is a single (scalar) output quantity and $\tilde{x}$ represents the N input quantities (x_1, x_2, …, x_n). The method of uncertainty propagation used in this work is the method of Monte Carlo simulation (MCS). The Monte Carlo simulation is a methodology which allows to make use of deterministic analysis in context of stochastic analysis. If all input model parameters (x_1, x_2, ..., x_n) are described by a probability density functions (PDF), an algorithm can be used to generate a input vector $x_i = [x_{i1}, x_{i2}, ..., x_{in}]^T$. Each element x_i of this vector should be generated in agreement with its PDFs. Applying the generated vector x_i to the equation (9), the corresponding output y_i is obtained. If this simulation process is repeated M times ($M >> 1$), the output is a vector $[y_1, y_2, ..., y_M]^T$ that can be considered as a sample answer population. Through this sample answer population system, a PDF can be determined and describes the behavior of the answer system [12], figure 8.

5.2 Uncertainty analysis in the material characterization process

The main uncertainty sources of creep test are related with the determination of specimen geometry, applied load, control of the environmental conditions (temperature, humidity and vibrations), material characteristics (material cracks, orthotropic material characteristic, aging

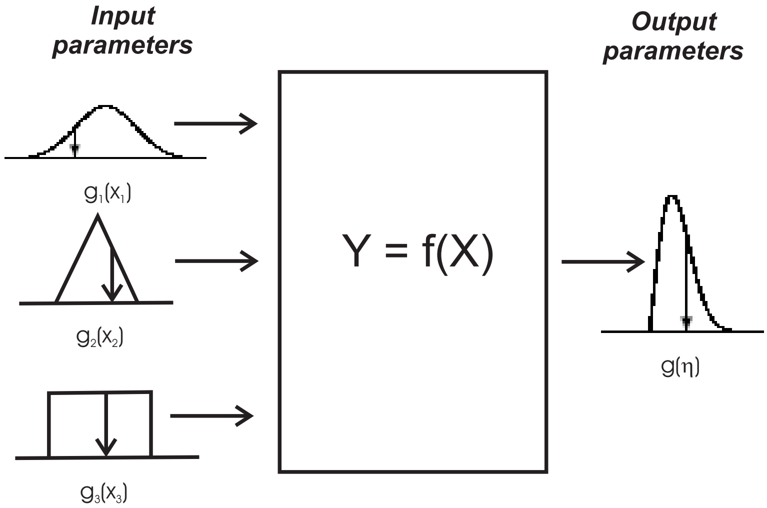

Figure 8: Illustration of the propagation of distributions by Monte Carlo simulation method.

effects, residual stress, etc.), test procedure, test apparatus limitations and curve-fitting process (figure 9).

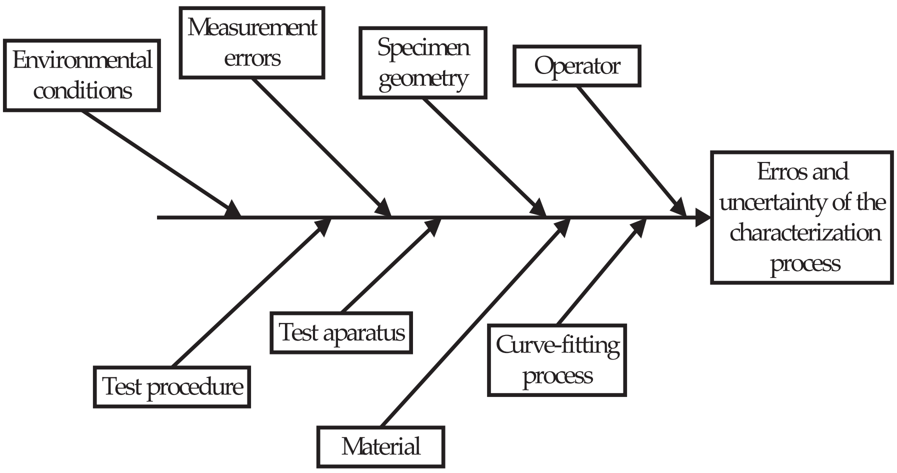

Figure 9: The main uncertainty sources of characterization process.

5.3 Uncertainty associated with fitted parameters

The material parameters are obtained from a nonlinear curve-fitting process. The influence of measurement uncertainty deformation (using strain gages) is incorporated into the material parameters uncertainty.

In agreement with references [4,5], the uncertainty (covariance) matrix associated with fitted parameters is approximated by

$$C = \sigma^2 \left(J^T J\right)^{-1} \tag{10}$$

where σ^2 the measured points variance and J Jacobian matrix evaluated in the solution x^*. The standard uncertainties associated with fitted parameters $u(a_j) = (C(j,j))^{1/2}$, i.e., the square roots of the diagonal elements of the uncertainty matrix C. If an estimate a priori of σ is not available, then the variance can be esteemed for

$$\hat{\sigma} = \frac{\|f\|}{\sqrt{n-m}} \tag{11}$$

where m is the number of adjusted parameters, $\|f\|$represents the norm of the residue and n is the number of measured points.

The tables 1 and 2 present the values and uncertainty levels of adjusted parameters for Maxwell model with 11 parameters and fractional Zener model, respectively.

Table 1: Classic material parameters obtained by nonlinear curve-fitting.

Parameters	**Mean**	**Standard deviation ($U_{68\%}$)**	$U_{68\%}/mean$	**Units**
J_0	3.228424e-010	3.510898e-012	0.011	1/GPa
J_1	9.347476e-012	1.349546e-013	0.014	1/GPa
J_2	3.544042e-011	5.686401e-013	0.016	1/GPa
J_3	5.227999e-011	6.131936e-013	0.012	1/GPa
J_4	5.755409e-011	4.993699e-013	0.009	1/GPa
J_5	1.399994e-010	5.534624e-013	0.004	1/GPa
τ_1	229.55	8.75	0.038	s
τ_2	5228.56	130.73	0.025	s
τ_3	50846.75	4355.18	0.019	s
τ_4	501810.54	4355.18	0.009	s
τ_5	2002253.33	5702.50	0.003	s

5.4 Uncertainty analysis in the interconversion process

Finite element programs used for packaging simulations usually require as input parameters the shear relaxation modulus $G(t)$ and the bulk relaxation modulus, $K(t)$. In this work we chose to measure the creep modulus $J(t)$ and the Poisson ratio v. Therefore, for using the commercial software Ansys in this work, it was necessary to determine $G(t)$ and $K(t)$ having measured $J(t)$ and v. It was done by the interconversion of data. The interconversion process is realized in two

Table 2: Fractionary material parameters obtained by nonlinear curve-fitting.

Parameters	**Mean**	**Standard deviation** ($U_{68\%}$)	$U_{68\%}/mean$	**Units**
E_0	3.330000e+009	5.257390e+006	0.002	1/GPa
E_1	2.728680e+008	5.257390e+006	0.008	1/GPa
α	2.531403e-001	9.451538e-004	0.004	
τ	6.985677e+010	1.141798e+007	0.000	1/s

steps. In the first step the relaxation modulus was obtained by having $J(t)$ by the creep test. The relaxation modulus and the creep compliance are connected by a simple relation between their Laplace transforming

$$\bar{E}(s) = \frac{1}{s^2 \bar{J}(s)} \tag{12}$$

where s is the Laplace Parameter, $\bar{E}(s)$ and $\bar{J}(s)$ are the Laplace transformation of relaxation modulus and creep compliance, respectively. An inverse Laplace transform of equation (12) can be done transforming it in a partial fraction expansion. The second step consisted in obtaining the shear relaxation modulus and bulk relaxation modulus by the application of the correspondence principle in the elastic relations

$$G = \frac{E}{2\,(1+v)} \tag{13}$$

and

$$K = \frac{E}{3(1-2v)}. \tag{14}$$

The aplication of the correspondence principle results in the Laplace equations

$$s\bar{G}\,(s) = \frac{s\bar{E}\,(s)}{2\,(1+s\bar{v}\,(s))} \Leftrightarrow \bar{G}\,(s) = \frac{s\bar{E}\,(s)}{2s\left(1+s\frac{v_0}{s}\right)} = \frac{\bar{E}\,(s)}{2\,(1+v_0)} \tag{15}$$

$$s\bar{K}\,(s) = \frac{s\bar{E}\,(s)}{3s(1-2s\bar{v}\,(s))} \Leftrightarrow \bar{K}\,(s) = \frac{s\bar{E}\,(s)}{3s(1-2s\frac{v_0}{s})} = \frac{\bar{E}\,(s)}{3(1-2v_0)} \tag{16}$$

The inverse Laplace transformation of equations (15) and (16) was obtained directly due to the elimination of the Laplace parameter. Therefore, the shear relaxation modulus $G(t)$

$$G\,(t) = G_{\text{inf}} - \sum_{i=1}^{n} G_i e^{-t/\tau_i} \tag{17}$$

and bulk relaxation modulus

$$K(t) = K_{\text{inf}} + \sum_{i=1}^{n} K_i e^{-t/\tau_i} \tag{18}$$

are given in the form of a Prony series, where G_{inf}, K_{inf}, K_i, G_i, and τ_i are material parameters.

The parameters uncertainties of creep compliance and Poisson's ratio, will be propagated through the interconversion process to the parameters of bulk and shear modulus.

The uncertainty propagation through the interconversion process is accomplished using the method of Monte Carlo simulation. The interconversion process is repeated many times in order to compose the uncertainty of bulk and shear modulus. The table 3 shows the values of bulk and shear modulus of PVC to 30 C obtained by the interconversion process.

Table 3: Bulk and Shear parameter's model obtained by interconversion process.

Parameters	**Mean**	**Standard deviation** ($U_{68\%}$)	$U_{68\%}/mean$	**Units**
G_0	1.155785e+009	1.343652e+007	0.012	1/GPa
G_1	3.850255e+007	1.147109e+006	0.030	1/GPa
G_2	9.850195e+007	2.530226e+006	0.026	1/GPa
G_3	1.368824e+008	2.897019e+006	0.021	1/GPa
G_4	1.198994e+008	2.280253e+006	0.019	1/GPa
G_5	1.578810e+008	2.127442e+006	0.013	1/GPa
K_0	3.228412e+009	7.905564e+007	0.024	1/GPa
K_1	1.075481e+008	3.964962e+006	0.037	1/GPa
K_2	2.751401e+008	9.172887e+006	0.033	1/GPa
K_3	3.823500e+008	1.159703e+007	0.030	1/GPa
K_4	3.349108e+008	9.629245e+006	0.029	1/GPa
K_5	4.410045e+008	1.126036e+007	0.026	1/GPa
τ_1	412.49	12.36	0.03	s
τ_2	4535.57	139.91	0.03	s
τ_3	43715.17	3302.96	0.02	s
τ_4	436104.08	3302.96	0.01	s
τ_5	1564712.48	4670.10	0.00	s

5.5 Uncertainty propagation through simulation process

The method of Monte Carlo simulation was used to propagate the input parameters uncertainties through the simulation model. The finite element commercial software ANSYS possesses the "Probabilistic Design System" tool, which uses the MCS to propagate the input parameters uncertainty through the finite element model of component. There is a possibility of creating a metamodel (i.e., a "model of the model") to the problem by the response surface method (RSM), which allows an analysis by Monte Carlo simulation computationally more efficient.

The simulation through the finite element method is a nonlinear problem. Therefore, the analysis with high number of degree-of-freedom through MCS requests a high computational time because MCS needs to repeat the problem simulation several times. One of the contributions in this work is the alternatives development to make possible the uncertainty propagation of viscoelastic structural problems through MCS. The main proposed solutions for the problem are the use the response surface method (RSM), the analysis using the models supplied for the classic, and the numeric correspondence principle associated to MCS. The case study presented will demonstrate the proposal methodology.

6 Case study: strain analysis of a viscoelastic cantilever beam

This case study is about uncertainty analysis involved in the strain simulation for a certain point in the surface of viscoelastic cantilever beam subjected to a constant load in its end (figure 10). The three-dimensional finite element model, used in the solution by PDS tool, is presented in the figure 11.

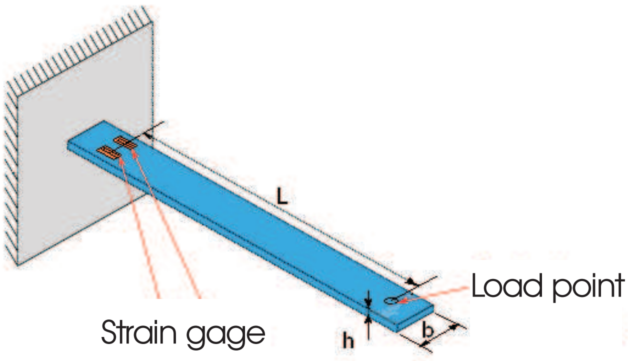

Figure 10: Viscoelastic cantilever beam.

An experimental test was realized to compare the experimental results with the value simulated using the correspondence principle and finite element method. The beam is subject to a constant load due to the mass applied in its end. Due to the viscoelastic material behavior, the beam will be deformed continually. Four strain gages were used in the beam (two strain gauges in the tensile surface and two strain gauges in the compression surface). The figure 12 shows the experimental apparatus. The figure 13 presents the experimental and simulated results.

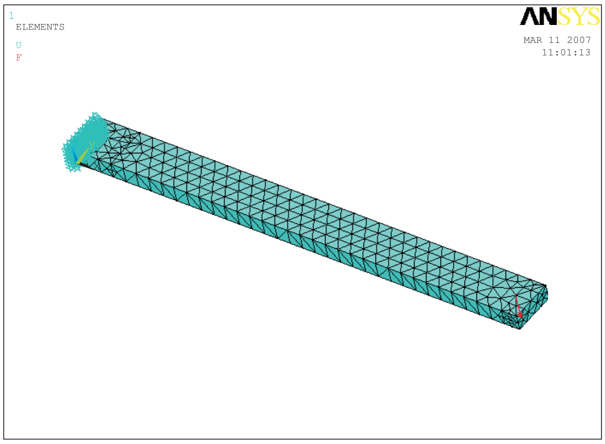

Figure 11: Finite element model of the cantilever beam.

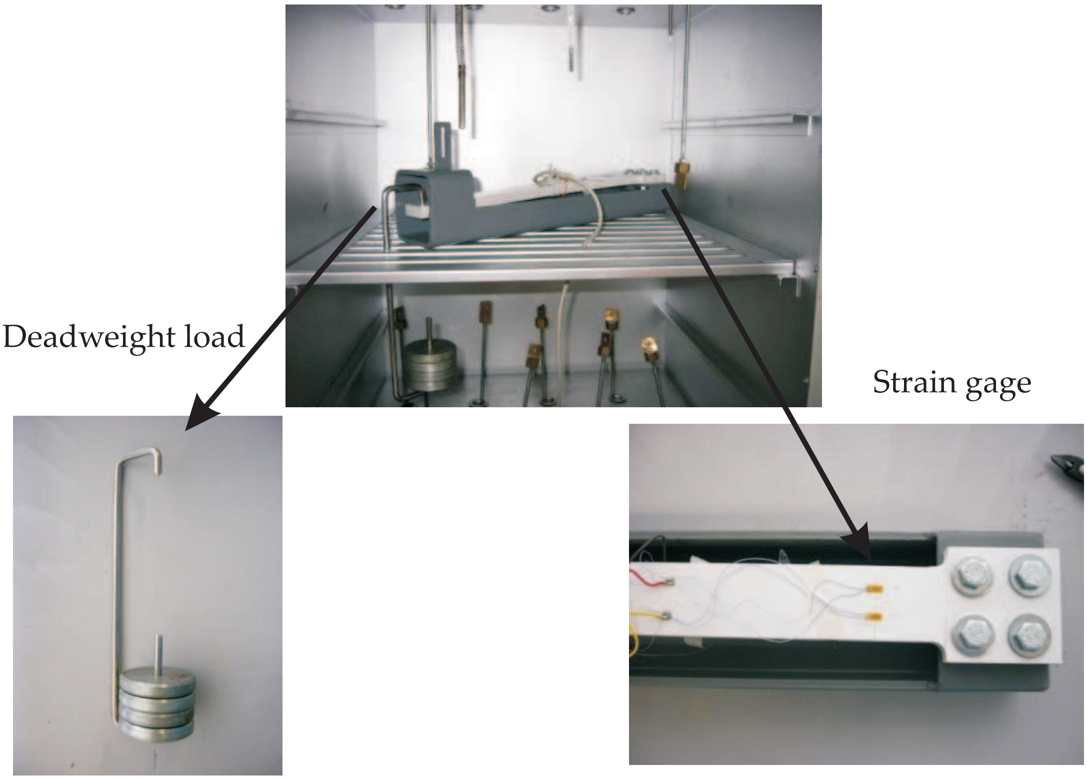

Figure 12: Apparatus used in the experimental test.

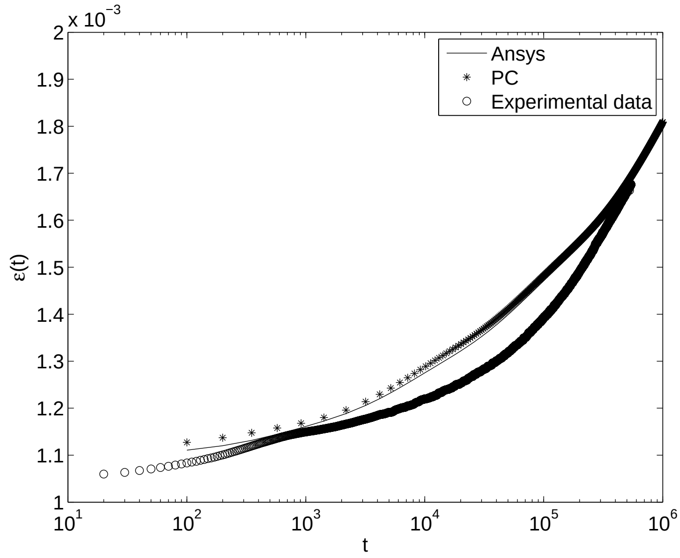

Figure 13: Deformation in the viscoelastic cantilever beam obtained experimentally and numerically via correspondence principle and finite element method.

6.1 Modeling aspects

The Monte Carlo simulation method, associated with the model supplied by the classic correspondence principle, requests the determination of the elastic analytic solution. The strain in a cantilever beam subjects the a load at the end is given by the equation

$$\varepsilon(x) = \frac{6F}{bh^2E}(L-x) \tag{19}$$

where L length of beam, x point of measured the deformation, E elastic modulus, b and h width and thickness of beam, respectively. For a step function load history, the solution of a viscoelastic problem by the classic correspondence principle is given for

$$\varepsilon(x,t) = \frac{6F}{bh^2}(L-x)J(t). \tag{20}$$

6.2 Input parameters characterization

6.2.1 Uncertainty associated with a determination of geometrical parameters

This study needs to determine the length, width and thickness of the transverse section beam. The main uncertainty sources acting in the determination of the geometry of beam are:

- Resolution of measurement systems (measurement scale and micrometer);
- Inherited uncertainty of calibrated measurement systems;

- Repeatability of measurement results;
- Uncertainty in determination of the PVC thermal coefficient, used for the uncertainty quantification due the material thermal dilation effects;
- The own beam geometry.

The combined uncertainty of measured geometry parameters on test body is computed as:

$$u_c = \sqrt{u_R^2 + u_{Re}^2 + u_{Cal}^2 + u_{\Delta T}^2} \tag{21}$$

where u_c combined standard uncertainty, u_R uncertainty due the instrument resolution, u_{Re} uncertainty due the repeatability of measurement results, u_{cal} inherited uncertainty of instrument calibration and $u_{\Delta T}$ the uncertainty due the temperature variation of stove.

The tables 4 and 5 present the main uncertainty sources and uncertainty budget for width and thickness obtained through the use of a DIGIMESS micrometer with resolution of 10 μm. It's presented in the table 6 the uncertainty budget of measurement beam length through the use of a measuring scale with resolution of 10 μm.

Table 4: Uncertainty budget associated with the determination of the width

	Uncertainty source	**Distribution**	**Value**	**Divider**	$u_{68\%}$	u_{eff}
R	Resolution (mm)	Rectangular	0.010	$2\sqrt{3}$	0.003	∞
Re	Repeatibility (mm)	Normal	0.006	1	0.006	16
Cal	Calibration erros (mm)	Normal	0.002	2	0.001	72
dT	Thermal dilatation (mm)	Rectangular	0.013	$\sqrt{3}$	0.007	∞
$\mathbf{u}_c$	Combined uncertainty (mm)	Normal			0.01	132

Table 5: Uncertainty budget associated with the determination of the thickness.

	Uncertainty source	**Distribution**	**Value**	**Divider**	$u_{68\%}$	u_{eff}
R	Resolution (mm)	Rectangular	0.010	$2\sqrt{3}$	0.003	∞
Re	Repeatibility (mm)	Normal	0.047	1	0.047	7
Cal	Calibration erros (mm)	Normal	0.002	2	0.001	72
dT	Thermal dilatation (mm)	Rectangular	0.034	$\sqrt{3}$	0.020	∞
$\mathbf{u}_c$	Combined uncertainty (mm)	Normal			0.05	10

6.2.2 Uncertainty associated with the applied load

The load is applied through a deadweight. The main uncertainty sources in the applied load quantification are:

Table 6: Uncertainty budget associated with the determination of the lenght.

	Uncertainty source	**Distribution**	**Value**	**Divider**	$u_{68\%}$	u_{eff}
R	Resolution (mm)	Rectangular	0.010	$2\sqrt{3}$	0.003	∞
Re	Repeatibility (mm)	Normal	0.060	1	0.060	7
Cal	Calibration erros (mm)	Normal	0.002	2	0.001	84
dT	Thermal dilatation (mm)	Rectangular	0.356	$\sqrt{3}$	0.205	∞
$\mathbf{u}_c$	Combined uncertainty (mm)	Normal			0.2	1434

- The misalignment in the load application;
- Uncertainty associated with gravitational acceleration;
- Resolution balance;
- Repeatability of measurement results of applied mass ;
- Inherited uncertainty of calibrated balance.

The table (7) presents the uncertainty budget of the applied mass determination. The table (8) shows the input parameters uncertainty of the beam simulation model.

Table 7: Uncertainty budget associated with the determination of the mass applied the beam.

	Uncertainty source	**Distribution**	**Value**	**Divider**	$u_{68\%}$	u_{eff}
R	Resolution (g)	Rectangular	1.0	$2\sqrt{3}$	0.3	∞
Re	Repeatibility (g)	Normal	0.2	1	0.2	10
Cal	Calibration erros (g)	Normal	0.5	2	0.3	89
$\mathbf{u}_c$	Combined uncertainty (g)	Normal			0.4	40

Table 8: Random input parameters for simulation model.

	Uncertainty source	**Distribution**	**Mean**	$u_{68\%}$	$u_{68\%}/mean$
m	Mass applied (g)	Normal	874.64	0.43	0.000
b	Width (mm)	Normal	32.18	0.05	0.002
h	Tickness (mm)	Normal	12.11	0.01	0.001
L	Legth (mm)	Normal	338.83	0.21	0.001
ν	Poisson ratio	Rectangular	0.38	0.01	0.029
g	Gravitational acceleration (m/s^2)	Rectangular	9.80665	0.00001	0.00000

6.3 Uncertainty propagation through simulation model

The table 9 shows the results by method of Monte Carlo simulation using tree simulation model: (a) model supplied by the classic correspondence principle (PC+SMC); (b) model supplied by numeric correspondence principle (SMC+PCN); (c) model supplied by the Response Surface Method (RSM). The table (10) shows the results through the method of simulation of Monte Carlo by the fractionary material model using the model supplied by PC and PCN.

Table 9: Obtained random output parameter for a classic material model.

Method	**Mean**	$u_{68\%}$	$u_{68\%}/mean$	**Realizations**
PC+SMC	1.09e-003	1.25e-005	0.012	1000
PDS	1.13e-003	1.32e-005	0.012	1000
PCN+SMC	1.17e-003	1.28e-005	0.011	1000
RSM	1.16e-003	1.30e-005	0.011	50000

Table 10: Obtained random output parameter for a fractionary material model.

Method	**Mean**	$u_{68\%}$	$u_{68\%}/mean$	**Realizations**
PC+SMC	1.03e-003	1.67e-006	0.002	1000
PCN+SMC	1.03e-003	3.42e-006	0.003	1000

7 Sensibility analysis

The sensibility analysis was accomplished using the Spearman rank order correlation coefficients [11]. It is presented in the figures (14) and (15) the result for a classic and fractional material model using the classic correspondence principle associated to the MCS. The objective of the sensibility analysis is present the parameters that have a larger contribution in the uncertainty of output model.

8 Conclusions

The probabilistic design is a technique for determining the effects of the uncertainties of the input parameters in the response of the simulation model. This technique allows the determination of the extent to which the uncertainties of the input parameters affect the results of an analysis by finite elements, for example. One of the shortcomings of the current probabilistic analysis is not to incorporate metrological aspects for the characterization of the stochastic behavior of the input variables were obtained by the measurement processes. Therefore, the main objective

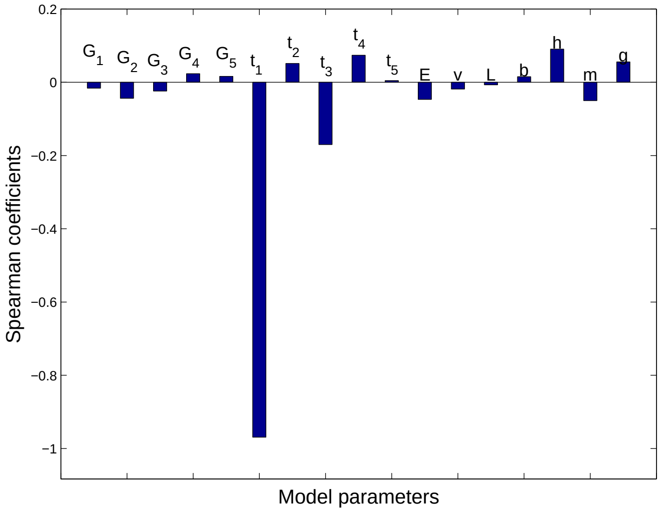

Figure 14: Spearman rank order correlation coefficients by solution obtained MCS associated the classic correspondence principle for a classic material model.

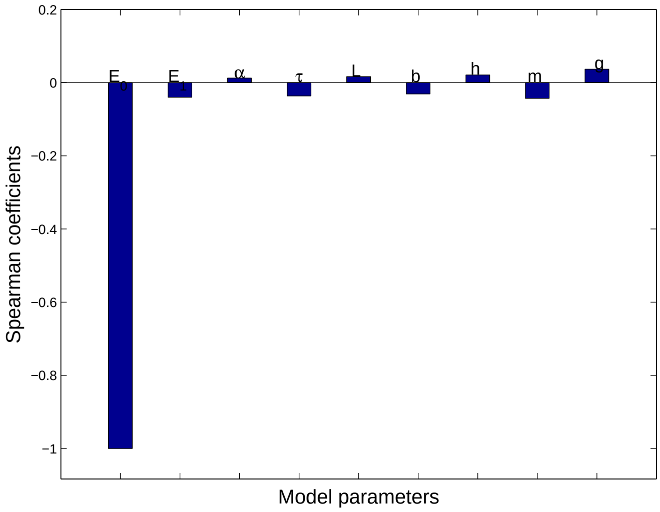

Figure 15: Spearman rank order correlation coefficients by solution obtained MCS associated the classic correspondence principle for a fractionary material model.

of this article was to present a methodology for the incorporation of metrological aspects to characterize the input parameters uncertainty of a model of structural simulation, which were obtained through measurement processes. The main tools used were the guidelines established by the Guide to the Expression of Uncertainty of Measurement - GUM [16], the use of the correspondence principle and the finite element method for modeling the structural problem in viscoelastic materials and the use of the Monte Carlo simulation method, to propagate the uncertainties.

The case study presented sought to exemplify the whole process of analysis, quantification and propagation of measurement uncertainty sources in an experiment, seeking to evaluate the dispersion of the model response and to classify the importance of each uncertainty source. The analysis of uncertainties along with an analysis of sensitivity enable the determination of the uncertainty sources that should first be minimized so that the model response be within the tolerances level set in the design of the structural component. The main characteristic of this tool is the ability to manage the uncertainty levels of the experiment in a consistent manner.

The Monte Carlo simulation method is a very robust tool. However, because of the non-linearity characteristics of the structural viscoelastic solution, the use of this method requires a greater time for computer simulation because of the repeated simulations necessity. The use of models provided by the classical and numerical correspondence principle presented itself as an efficient alternative for the minimization of time required by the computer simulation method for the Monte Carlo simulation.

References

[1] ANSI/ASTM D 2991-71. Standard test methods for tensile, compressive, and flexural creep and creep-rupture of plastics. *Technical report*, 2001.

[2] ANSYS.

[3] R. L. Bagley and P. J. Torvik. On the fractional calculus model of viscoelastic behavior. *Journal of Rheology*, 30(1):133–155, 1986.

[4] M. G. Baker and M. G. Cox. Discrete modeling and experimental data analysis. *Technical report, National Physical Laboratory – NPL*, 2004.

[5] P. R. Bevington. *Data reduction and error analysis for the physical sciences.* McGraw-Hill Book Company, New York, 1969.

[6] R. M. Christensen. *Theory of Viscoelasticity, a Introduction.* Academic Press, New York, 2 edition, 1982.

[7] K. Diethelm, N. J. Ford, A. D. Freed, and Y. Luchko. Algorithms for the fractional calculus: A selection of numerical methods. *Comput. Methods Appl. Mech. Eng.*, 2000.

[8] M. Enelund and P. Olsson. Damping described by fading memory - analysis and application to fractional derivative models. *International Journal of Solids and Structures*, 36:939–970, 1999.

[9] W. N. Findley. Mechanism and mechanics of creep of plastics. *SPE Journal*, 16:57–65, 1960.

[10] W. Flugge. *Viscoelasticity.* Springer-Verlag, 2 edition, 1978.

[11] R. Gorenflo. Fractional calculus: Some numerical methods. In A. Carpinteri and F. Mainardi, editors, *Fractals and Fractional Calculus in Continuum Mechanics*, pages 277–290, Wien, 1997. Spring Verlag.

[12] GUM SUPPL 1. Guide to the Expression of Uncertainty in Measurement (GUM) – Supplement 1: Numerical Methods for the Propagation of Distributions". In accordance with the ISO/IEC Directives, Part 1, 2001, Clause A.5, this draft is submitted to the ISO national bodies for approval. International Organization for Standardization. 2004.

[13] R. C. Koeller. Applications of fractional calculus to the theory of viscoelasticity. *ASME Journal of Applied Mechanics*, 51(2):299–307, 1984.

[14] S. W. J. Welch, A. L. R. Rorrer, and R. G. Duren. Application of time-based fractional calculus methods to viscoelastic creep and stress relaxation of materials. *Mechanics of Time-Dependent Materials*, 3:279–303, 1999.

[15] O. C. Zienkiewicz and R. L. Taylor. *The finite element method*. McGraw Hill, 1991. Vol. I., 1989, Vol. II.