Latin American Journal of Solids and Structures
www.lajss.org

Numerical analysis of the propagation characteristics of Stoneley waves at an interface between microstretch thermoelastic diffusion solid half spaces

Abstract

This paper is concerned with the study of propagation of Stoneley waves at the interface of two dissimilar isotropic microstretch thermoelastic diffusion medium in the context of generalized theories of thermoelasticity. The dispersion equation of Stoneley waves is derived in the form of a determinant by using the boundary conditions. The dispersion curves giving the phase velocity and attenuation coefficients with wave number are computed numerically. Numerically computed results are shown graphically to depict the diffusion effect alongwith the relaxation times in microstretch thermoelastic diffusion solid half spaces for thermally insulated and impermeable boundaries, respectively. The components of displacement, stress, couple stress, microstress, and temperature change are presented graphically for two dissimilar microstretch thermoelastic diffusion half-spaces. Several cases of interest under different conditions are also deduced and discussed.

Keywords

Microstrecth, Dispersion equation, Stoneley waves, Propagation characteristics.

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Received 10.01.2014
In revised form 08.04.2014
Accepted 23.04.2014
Available online 09.10.2014

1 INTRODUCTION

The exact nature of the layers beneath the earth's surface is not known. One has, therefore to consider various appropriate models for the purpose of theoretical investigations. These problems not only provide better information about the internal composition of the earth but also helpful in exploration of valuable materials beneath the earth surface.

Mathematical modeling of surface wave propagation along the free boundary of an elastic half-space or along the interface between two dissimilar elastic half-spaces has been subject of continued interest for many years. These waves are well known in the study of geophysics, ocean acoustics, SAW devices and more recently nondestructive evaluation. The study of surface wave propagation is of much practical importance in various fields such as earthquake engineering, soil dynamics, aeronautics, nuclear reactors, high energy particle accelerator etc. Rayleigh (1885) discussed the surface wave propagation along the free boundary of an elastic half-space, non-attenuated in their direction of propagation and damped normal to the boundary.

Stoneley (1924) studied the existence of waves, which are similar to surface waves and propagating along the plane interface between two distinct elastic solid half spaces in perfect contact. Stoneley waves can also propagate on interfaces either two elastic media or a solid medium and a liquid medium. Stoneley (1924) derived the dispersion equation for the propagation of Stoneley waves. Tajuddin (1995) investigated the existence of Stoneley waves at an interface between two micropolar elastic half spaces.

Eringen (1966b, 1968) developed the theory of micromorphic bodies by considering a material point as endowed with three deformable directions. Subsequently, he developed the theory of microstretch elastic solid (1971) which is a generalization of micropolar elasticity (1966a). The material points in microstretch elastic body can stretch and contract independently of the translational and rotational processes. The difference between these solids and micropolar elastic solids stems from the presence of scalar microstretch and a vector first moment. These solids can undergo intrinsic volume change independent of the macro volume change and is accompanied by a non deviatoric stress moment vector.

Eringen(1990) also developed the theory of thermo microstretch elastic solids. The microstretch continuum is a model for Bravias lattice with a basis on the atomic level and a two phase dipolar solid with a core on the macroscopic level. For example, composite materials reinforced with chopped elastic fibres, porous media whose pores are filled with gas or inviscid liquid, asphalt or other elastic inclusions and 'solid-liquid' crystals, etc., should be characterizable by microstretch solids. A comprehensive review on the micropolar continuum theory has been given in his book by Eringen(1999).

Iesan and Pompei (1995) discussed the equilibrium theory of microstretch elastic solids. The problem of wave propagation through a cylindrical bore contained in a microstretch elastic medium has been studied by Kumar and Deswal(2002). Tomar and Singh (2006) discussed the propagation of Stoneley waves at an interface between two microstretch elastic half-spaces. Kumar and Pratap (2009) studied the propagation of free vibrations in microstretch thermoelastic homogeneous isotropic, thermally conducting plate bordered with layers of inviscid liquid on both sides subjected to stress free thermally insulated and isothermal conditions. Markov (2009) discussed the propagation of Stoneley elastic wave at the boundary of two fluid-saturated porous media and determined the velocity and attenuation of the Stoneley surface waves. Ahmed and Abo-Dahab (2012) studied the propagation of Rayleigh and Stoneley waves in a thermoelastic orthotropic granular half-space supporting a different layer under the influence of initial stress and gravity field.

Diffusion can be defined as the random walk to accumulate the particles from region of high concentration to that of low concentration. At the present time, there is a great deal of interest in the study of this phenomenon due to its application in geophysics and electronic industry. Study of phenomenon of diffusion is utilized to enhance the conditions of oil extraction (searching ways of more efficiently recovering oil from its deposits).

Nowacki (1974a, 1974b, 1974c, 1976) in a series of papers presented the theory of thermoelastic diffusion by using coupled thermoelastic model. Uniqueness and reciprocity theorems for the equations of generalized thermoelastic diffusion problem, in isotropic media, was proved by Sherief et al.(2004)on the basis of the variational principle equations, under restrictive assumptions on the elastic coefficients. Kumar and Kansal (2008) derived the basic equation of anisotropic thermoelastic diffusion based upon Green-Lindsay model and discussed the Lamb waves.

Kumar and Chawla (2009) discussed the wave propagation at the imperfect boundary between transversely isotropic themoelastic diffusive half space and an isotropic elastic layer. Kumar and Kansal(2011) construct the fundamental solution of system of differential equations in the theory of thermomicrostretch elastic diffusive solids in case of steady oscillations in terms of elementary functions. Sharma (2007, 2008) discussed the plane harmonic generalized thermoelastic diffusive waves and elasto thermodiffusive surface waves in heat-conducting solids. Recently Kumar et al. (2013)studied the reflection and transmission of plane waves at the interface between a microstretch thermoelastic diffusion solid half-space and elastic solid half space.

Keeping in view of these applications, dispersion equations for Stoneley waves at the interface of two dissimilar isotropic microstretch thermoelastic diffusion medium in the context of generalized theories of thermoelasticity have been derived. It is found that Stoneley waves in a microstretch thermoelastic diffusion solid medium are dispersive. Numerical computations are performed for a particular model to study the variations of phase velocity and attenuation coefficient with respect to wave number. The present work is novel and have not been discussed earlier in the literature. The results presented in this paper should prove useful for researchers in material science, designers of new materials as well as for those working on the development of theory of elasticity.

2 BASIC EQUATIONS

Following Eringen (1999), Sherief et al. (2004) and Kumar & Kansal (2008), the equations of motion and the constitutive relations in a homogeneous isotropic microstretch thermoelastic diffusion solid in the absence of body forces, body couples, stretch force, and heat sources are given by

$$
\begin{aligned}
&(\lambda+2\mu+K)\nabla(\nabla.\vec{u})-(\mu+K)\nabla\times\nabla\times\vec{u}+K\nabla\times\vec{\varphi}+\lambda_O\nabla\varphi^* \\
&\qquad -\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C=\rho\frac{\partial^2\vec{u}}{\partial t^2},
\end{aligned} \tag{1}
$$

$$
(\alpha+\beta+\gamma)\nabla(\nabla.\vec{\varphi})-\gamma\,\nabla\times(\nabla\times\vec{\varphi})+K\nabla\times\vec{u}-2K\vec{\varphi}=\rho j\frac{\partial^2\vec{\varphi}}{\partial t^2}, \tag{2}
$$

$$
\alpha_0\nabla^2\varphi^*+\nu_1\left(T+\tau_1\dot{T}\right)+\nu_2\left(C+\tau^1\dot{C}\right)-\lambda_1\varphi^*-\lambda_0\nabla.\vec{u}=\frac{\rho j_0}{2}\frac{\partial^2\varphi^*}{\partial t^2} \tag{3}
$$

$$
K^*\nabla^2T=\beta_1T_0\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\nabla.\dot{\vec{u}}+\nu_1T_0\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\dot{\varphi}^*+\rho C^*\left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{T}+aT_0\left(\dot{C}+\gamma_1\ddot{C}\right), \tag{4}
$$

$$D\beta_2\nabla^2(\nabla.\vec{u}) + D\nu_2\nabla^2\varphi^* + Da\nabla^2\left(T+\tau_1\dot{T}\right) + \left(\dot{C}+\varepsilon\tau^0\ddot{C}\right) - Db\nabla^2\left(C+\tau^1\dot{C}\right) = 0, \tag{5}$$

and constitutive relations are

$$t_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{i,j}+u_{j,i}\right) + K\left(u_{j,i}-\varepsilon_{ijr}\varphi_r\right) + \lambda_o\delta_{ij}\varphi^* - \beta_1(1+\tau_1\frac{\partial}{\partial t})T\delta_{ij} - \beta_2(1+\tau^1\frac{\partial}{\partial t})C\delta_{ij}, \tag{6}$$

$$m_{ij} = \alpha\varphi_{r,r}\delta_{ij} + \beta\varphi_{i,j} + \gamma\varphi_{j,i} + b_0\varepsilon_{mji}\varphi^*_{,m}, \tag{7}$$

$$\lambda_i^* = \alpha_0\varphi^*_{,i} + b_0\varepsilon_{ijm}\varphi_{j,m}, \tag{8}$$

where

$\lambda, \mu, \alpha, \beta, \gamma, K, \lambda_o, \lambda_1, \alpha_o, b_o,$ are material constants , ρ is the mass density , $\vec{u}=\left(u_1,u_2,u_3\right)$ is the displacement vector and $\vec{\varphi}=\left(\varphi_1,\varphi_2,\varphi_3\right)$ is the microrotation vector, φ^* is the scalar microstretch function, T and T_0 are the small temperature increment and the reference temperature of the body chosen such that $\left|T/T_0\right| \ll 1,$ C is the concentration of the diffusion material in the elastic body. K^* is the coefficient of the thermal conductivity, C^* the specific heat at constant strain, D is the thermoelastic diffusion constant. a, b are, respectively, coefficients describing the measure of thermodiffusion and of mass diffusion effects, $\beta_1=\left(3\lambda+2\mu+K\right)\alpha_{t1}$, $\beta_2=\left(3\lambda+2\mu+K\right)\alpha_{c1}$, $\nu_1=\left(3\lambda+2\mu+K\right)\alpha_{t2}$, $\nu_2=\left(3\lambda+2\mu+K\right)\alpha_{c2}$, α_{t1},α_{t2} are coefficients of linear thermal expansion and α_{c1},α_{c2} are the coefficients of linear diffusion expansion. j is the microintertia, j_o is the microinertia of the microelements, t_{ij} and m_{ij} are components of stress and couple stress tensors respectively, λ_i^* is the microstress tensor, $e_{ij}\left(=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right)\right)$ are components of infinitesimal strain, e_{kk} is the dilatation, δ_{ij} is the Kronecker delta, τ^0,τ^1 are diffusion relaxation times with $\tau^1 \geq \tau^0 \geq 0$ and τ_0,τ_1 are thermal relaxation times with $\tau_1 \geq \tau_0 \geq 0$. Here $\tau_0=\tau^0=\tau_1=\tau^1=\gamma_1=0$ for Coupled Thermoelasitc (CT) model, $\tau_1=\tau^1=0,\ \varepsilon=1, \gamma_1=\tau_0$ for Lord-Shulman (L-S) model and $\varepsilon=0,\ \gamma_1=\tau^0$ where $\tau^0>0$ for Green-Lindsay (G-L) model.

In the above equations, a comma followed by a suffix denotes spatial derivative and a superposed dot denotes the derivative with respect to time respectively.

3 FORMULATION OF THE PROBLEM

We consider two homogeneous isotropic microstretch generalized thermoelastic diffusion half-spaces M_1 and M_2 connecting at the interface $x_3=0$. The origin of the coordinate system (x_1,x_2,x_3) is taken at any point on the plane horizontal surface with x_3-axis, pointing vertically downward to the half-space, which is thus represented by $x_3 \geq 0$. We choose the x_1 axis in the direction of wave propagation in such a way that all the particles on a line parallel to the x_2 axis are equally displaced. Therefore, all field quantities are independent of the x_2 coordinate. Medium M_2 occupies the region $-\infty < x_3 < 0$ and the region $x_3 > 0$ is occupied by the half-space (medium M_1). The plane $x_3 = 0$ represents the interface between two media M_1 and M_2. For the two dimensional problem, we take

$$\vec{u}(x_1,x_3,t)=(u_1,0,u_3),\ \vec{\varphi}=(0,\varphi_2,0), \varphi^*(x_1,x_3,t), \mathrm{T}(x_1,x_3,t),\ C(x_1,x_3,t), \tag{9}$$

We define the following dimensionless quantities

$$\begin{aligned}
&\left(x_1^{'},x_3^{'}\right)=\frac{\omega^*}{c_1}\left(x_1,x_3\right),\left(u_1^{'},u_3^{'}\right)=\frac{\rho c_1\omega^*}{\beta_1 T_o}\left(u_1,u_3\right),\varphi_2^{'}=\frac{\rho c_1^2}{\beta_1 T_o}\varphi_2,\varphi^{*'}=\frac{\rho c_1^2}{\beta_1 T_o}\varphi^*,\\
&t_{ij}^{'}=\frac{t_{ij}}{\beta_1 T_o},m_{ij}^{'}=\frac{\omega^*}{c_1\beta_1 T_o}m_{ij},\lambda_i^{*'}=\frac{\lambda_i^*\omega^*}{c_1\beta_1 T_o},T^{'}=\frac{T}{T_o},C^{'}=\frac{\beta_2 C}{\rho c_1^2},t^{'}=\omega^* t,\\
&\tau_o^{'}=\omega^*\tau_o,\tau^{0'}=\omega^*\tau^0,\tau_1^{'}=\omega^*\tau_1,\tau^{1'}=\omega^*\tau^1,
\end{aligned} \tag{10}$$

where

$\omega^*=\dfrac{\rho C^* c_1^2}{K^*}$, $c_1^2=\dfrac{\lambda+2\mu+K}{\rho}$, ω^* is the characteristic frequency of the medium,

Upon introducing the quantities (10) in equations (1)-(5), with the aid of (9) and after suppressing the primes, we obtain

$$\delta^2\frac{\partial e}{\partial x_1}+\left(1-\delta^2\right)\nabla^2 u_1-a_1\frac{\partial\varphi_2}{\partial x_3}+a_2\frac{\partial\varphi^*}{\partial x_1}-\tau_t^1\frac{\partial T}{\partial x_1}-a_3\tau_c^1\frac{\partial C}{\partial x_1}=\frac{\partial^2 u_1}{\partial t^2}, \tag{11}$$

$$\delta^2\frac{\partial e}{\partial x_3}+\left(1-\delta^2\right)\nabla^2 u_3+a_1\frac{\partial\varphi_2}{\partial x_1}+a_2\frac{\partial\varphi^*}{\partial x_3}-\tau_t^1\frac{\partial T}{\partial x_3}-a_3\tau_c^1\frac{\partial C}{\partial x_3}=\frac{\partial^2 u_3}{\partial t^2}, \tag{12}$$

$$a_4\nabla^2\varphi_2+a_5\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)-a_6\varphi_2=\frac{\partial^2\varphi_2}{\partial t^2}, \tag{13}$$

$$\left(\delta_1^2\nabla^2-a_7\right)\varphi^*-a_8 e+a_9\tau_t^1 T+a_{10}\tau_c^1 C=\frac{\partial^2\varphi^*}{\partial t^2}, \tag{14}$$

$$\nabla^2 T=a_{11}\tau_e^0\frac{\partial e}{\partial t}+a_{12}\tau_e^0\frac{\partial\varphi^*}{\partial t}+\tau_t^0\frac{\partial T}{\partial t}+a_{13}\tau_c^0\frac{\partial C}{\partial t}, \tag{15}$$

$$a_{14}\nabla^2 e+a_{21}\nabla^2\varphi^*+a_{15}\tau_t^1\nabla^2 T+\tau_f^0\frac{\partial C}{\partial t}-a_{16}\tau_c^1\nabla^2 C=0, \tag{16}$$

where

$$\left(a_1,a_2\right)=\frac{1}{\rho c_1^2}\left(K,\lambda_0\right),a_3=\frac{\rho c_1^2}{\beta_1 T_0},\left(a_4,a_5,a_6\right)=\frac{1}{j\rho}\left(\frac{\gamma}{c_1^2},\frac{K}{\omega^{*2}},\frac{2K}{\omega^{*2}}\right),\delta^2=\frac{\lambda+\mu}{\rho c_1^2}$$

$$\left(a_{11},a_{12},a_{13}\right)=\frac{1}{K^*\omega^*}\left(\frac{T_0\beta_1^2}{\rho},\frac{\beta_1T_0\nu_1}{\rho},\frac{\rho c_1^4a}{\beta_2}\right),\left(a_{14},a_{15},a_{16}\right)=\frac{D\omega^*}{c_1^2}\left(\frac{\beta_1^2}{\rho c_1^2},\frac{\beta_2a}{\beta_1},b\right)$$

$$\left(a_7,a_8,a_9,a_{10}\right)=\frac{2}{j_0\omega^{*2}}\left(\frac{\lambda_1}{\rho},\frac{\lambda_0}{\rho},\frac{\nu_1c_1^2}{\beta_1},\frac{\nu_2\rho c_1^4}{\beta_1\beta_2T_0}\right),\delta_1^2=\frac{c_2^2}{c_1^2},c_2^2=\frac{2\alpha_0}{\rho j_0},a_{21}=\frac{D\nu_2\beta_2\omega^*}{\rho c_1^4}$$

$$\tau_t^1=1+\tau_1\frac{\partial}{\partial t},\tau_c^1=1+\tau^1\frac{\partial}{\partial t},\tau_f^0=1+\varepsilon\tau^0\frac{\partial}{\partial t},\tau_t^0=1+\tau_0\frac{\partial}{\partial t},$$

$$\tau_e^0=1+\varepsilon\tau_0\frac{\partial}{\partial t},\tau_c^0=1+\gamma_1\frac{\partial}{\partial t},e=\frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3},\nabla^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}$$

We introduce the potential functions ϕ *and* ψ through the relations

$$u_1=\frac{\partial\phi}{\partial x_1}-\frac{\partial\psi}{\partial x_3},u_3=\frac{\partial\phi}{\partial x_3}+\frac{\partial\psi}{\partial x_1}, \tag{17}$$

in the equations (11)-(16), we obtain

$$\nabla^2\phi+a_2\varphi^*-\tau_t^1T-a_3\tau_c^1C=\ddot{\phi}, \tag{18}$$

$$\left(1-\delta^2\right)\nabla^2\psi+a_1\varphi_2=\ddot{\psi}, \tag{19}$$

$$\left(a_4\nabla^2-a_6\right)\varphi_2-a_5\nabla^2\psi=\ddot{\varphi}_2, \tag{20}$$

$$\left(\delta_1^2\nabla^2-a_7\right)\varphi^*-a_8\nabla^2\phi+a_9\tau_t^1T+a_{10}\tau_c^1C=\ddot{\varphi}^*, \tag{21}$$

$$\nabla^2T=\tau_e^0\left(a_{11}\nabla^2\dot{\phi}+a_{12}\dot{\varphi}^*\right)+\tau_t^0\dot{T}+a_{13}\tau_c^0\dot{C}, \tag{22}$$

$$a_{14}\nabla^4\phi+a_{21}\nabla^2\varphi^*+a_{15}\tau_t^1\nabla^2T-a_{16}\tau_c^1\nabla^2C+\tau_f^0\dot{C}=0 \tag{23}$$

4 SOLUTION OF THE PROBLEM

We assume the solutions of the form

$$\left\{\phi,\varphi^*,T,C\right\}\left(x_1,x_3,t\right)=\left\{\tilde{\phi},\tilde{\varphi}^*,\tilde{T},\tilde{C}\right\}\left(x_3\right)e^{i\xi(x_1-ct)} \tag{24}$$

where ξ is the wave number, $\omega=\xi c$ is the angular frequency, and c is phase velocity of the wave. Using (24) in equations (18), (21)–(23) and satisfying the radiation condition $\phi,\varphi^*,T,C\to 0$ as $x_3\to\infty$, we obtain the values of ϕ,φ^*,T,C for medium M_1,

$$\left\{\phi,\varphi^*,T,C\right\}=\sum_{p=1}^{4}\left[A_p\left\{1,n_{1p},n_{2p},n_{3p}\right\}e^{-m_p x_3}\right]e^{i\xi(x_1-ct)} \tag{25}$$

where $A_p,(p=1,2,3,4)$ are arbitrary constants, the coupling constants n_{1p},n_{2p},n_{3p} given in appendix B.

We attach bar for the variables in the medium M_2 and write the appropriate values of $\bar{\phi},\bar{\varphi}^*,\bar{T},\bar{C}$ for $M_2(x_3<0)$ satisfying the radiation conditions as

$$\left\{\bar{\phi},\bar{\varphi}^*,\bar{T},\bar{C}\right\}=\sum_{p=1}^{4}\left[\bar{A}_p\left\{1,\bar{n}_{1p},\bar{n}_{2p},\bar{n}_{3p}\right\}e^{\bar{m}_p x_3}\right]e^{i\xi(x_1-ct)} \tag{26}$$

where

m_p^2 $(p=1,2,3,4)$ are the roots of the equation

$$D^8+A_1^*D^6+B_1^*D^4+C_1^*D^2+D_1^*=0 \tag{27}$$

and

$\bar{m}_p^2$ $(p=1,2,3,4)$ are the roots of the equation

$$D^8+\bar{A}_1^*D^6+\bar{B}_1^*D^4+\bar{C}_1^*D^2+\bar{D}_1^*=0 \tag{28}$$

where $D=d/dx_3$, the coefficients $A_1^*,B_1^*,C_1^*,D_1^*,\bar{A}_1^*,\bar{B}_1^*,\bar{C}_1^*$ and $\bar{D}_1^*$ are given in appendix A,

Similarly, we assume the solutions of the field equations as

$$\left\{\psi,\varphi_2\right\}\left(x_1,x_3,t\right)=\left\{\tilde{\psi},\tilde{\varphi}_2\right\}\left(x_3\right)e^{i\xi(x_1-ct)} \tag{29}$$

Using (29) in equations (19) and (20), and satisfying the radiation condition $\psi,\varphi_2\to 0$ as $x_3\to\infty$, we obtain the values of ψ,φ_2 for medium M_1,

$$\left\{\psi,\varphi_2\right\}=\sum_{p=5}^{6}\left[A_p\left\{1,n_{4p}\right\}e^{-m_p x_3}\right]e^{i\xi(x_1-ct)} \tag{30}$$

where A_p $(p=5,6)$ are arbitrary constants, n_{4p} is the coupling constant given in appendix B.

We attach bar for the variables in the medium M_2 and write the appropriate values of $\bar{\psi},\bar{\varphi}_2$ for $M_2(x_3<0)$ satisfying the radiation conditions as

$$\left\{\bar{\psi},\bar{\varphi}_2\right\}=\sum_{p=5}^{6}\left[\bar{A}_p\left\{1,\bar{n}_{4p}\right\}e^{\bar{m}_p x_3}\right]e^{i\xi(x_1-ct)} \tag{31}$$

where

m_p^2 $(p=5,6)$ are the roots of the equation

$$D^4+A_2^*D^2+B_2^*=0 \tag{32}$$

and

$\bar{m}_p^2\ (p=5,6)$ are the roots of the equation

$$D^4+\bar{A}_2^* D^2+\bar{B}_2^*=0 \tag{33}$$

where $D=d/dx_3$, the coefficients $A_2^*, B_2^*, \bar{A}_2^*$ and $\bar{B}_2^*$ are given in appendix A,
The roots of equation (27) in the descending order corresponds to the velocities of propagation of four possible waves, namely longitudinal displacement wave (LD), thermal wave (T), mass diffusion wave (MD) and longitudinal microstretch wave (LM), respectively. Similarly, two roots of the equation (32) in the descending order corresponds to the velocities of propagation of two coupled transverse displacement and transverse microrotational waves (CD I, CD II), respectively.

(i) In the absence of diffusion effect, equation (27) leads to sixth order differential equation

$$D^6+A_3^* D^4+B_3^* D^2+C_3^*=0, \text{ where } A_3^*, B_3^* \text{ and } C_3^* \text{ are given in appendix A} \tag{34}$$

The roots of the equation (34) $m_p^2\ (p=1,2,3)$ correspond to the LD, T and LM waves, respectively.
Clearly, we can notice here that, on neglecting the diffusion effect, the wave corresponding to this parameter namely mass diffusion wave (MD) become deceased.
Therefore, it is observed from the equation (27) and (34), that there exist a new type of wave namely MD wave.

(ii) On neglecting the diffusion, micropolarity and microstretch effects, equation (27) and (32) simultaneously leads to the forth and second order differential equations as

$$D^4+A_4^* D^2+B_4^*=0, \tag{35}$$

and

$$D^2+\frac{b_{26}}{\left(1-\delta^2\right)}=0, \tag{36}$$

where A_4^* and B_4^* are given in appendix A.
The roots of the equation (35) correspond to the Longitudinal wave (P-wave), and T waves, and (36) relate to the SV- wave, respectively.
Therefore, it is again observed that there exist new type of wave in (34) namely Longitudinal microstretch wave (LM) and transverse microrotational waves (CD II) in (32) which become decoupled in this case .
Substituting the values of $\phi, \psi, \bar{\phi}$ and $\bar{\psi}$ from equations (25), (26), (30) and (31) in equation (17), we obtained displacement components
For medium M_1

$$u_1=\left[i\xi\left(A_1e^{-m_1x_3}+A_2e^{-m_2x_3}+A_3e^{-m_3x_3}+A_4e^{-m_4x_3}\right)+m_5A_5e^{-m_5x_3}+m_6A_6e^{-m_6x_3}\right]e^{i\xi(x_1-ct)},$$

$$u_3=\left[-\left(m_1A_1e^{-m_1x_3}+m_2A_2e^{-m_2x_3}+m_3A_3e^{-m_3x_3}+m_4A_4e^{-m_4x_3}\right)+i\xi\left(A_5e^{-m_5x_3}+A_6e^{-m_6x_3}\right)\right]e^{i\xi(x_1-ct)},$$

For medium M_2

$$\bar{u}_1 = \left[i\xi \left(\bar{A}_1 e^{\bar{m}_1 x_3} + \bar{A}_2 e^{\bar{m}_2 x_3} + \bar{A}_3 e^{\bar{m}_3 x_3} + \bar{A}_4 e^{\bar{m}_4 x_3} \right) - \left(\bar{m}_5 \bar{A}_5 e^{\bar{m}_5 x_3} + \bar{m}_6 \bar{A}_6 e^{\bar{m}_6 x_3} \right) \right] e^{i\xi(x_1 - ct)},$$

$$\bar{u}_3 = \left[\left(\bar{m}_1 \bar{A}_1 e^{\bar{m}_1 x_3} + \bar{m}_2 \bar{A}_2 e^{\bar{m}_2 x_3} + \bar{m}_3 \bar{A}_3 e^{\bar{m}_3 x_3} + \bar{m}_4 \bar{A}_4 e^{\bar{m}_4 x_3} \right) + i\xi \left(\bar{A}_5 e^{\bar{m}_5 x_3} + \bar{A}_6 e^{\bar{m}_6 x_3} \right) \right] e^{i\xi(x_1 - ct)}.$$

5 BOUNDARY CONDITIONS

The following boundary conditions must be satisfied on the boundary between two microstretch thermoelastic diffusion media. Mathematically, these can be written (at the surface $x_3 = 0$) as

(i) $t_{33} = \bar{t}_{33}$, (37)

(ii) $t_{31} = \bar{t}_{31}$, (38)

(iii) $m_{32} = \bar{m}_{32}$, (39)

(iv) $\lambda_3^* = \bar{\lambda}_3^*$, (40)

(v) $u_3 = \bar{u}_3$, (41)

(vi) $u_1 = \bar{u}_1$, (42)

(vii) $\varphi_2 = \bar{\varphi}_2$ (43)

(viii) $\varphi^* = \bar{\varphi}^*$ (44)

(ix) $T = \bar{T}$ (45)

(x) $C = \bar{C}$ (46)

(xi) $K^* \left(\dfrac{\partial T}{\partial x_3} \right) = \bar{K}^* \dfrac{\partial \bar{T}}{\partial x_3}$, (47)

(xii) $\eta^* \dfrac{\partial C}{\partial x_3} = \bar{\eta}^* \dfrac{\partial \bar{C}}{\partial x_3}$, (48)

6 DERIVATIONS OF THE SECULAR EQUATIONS

Making use of equations (25)-(26), (30) and (31) in the equations (37)-(48), we obtain a system of twelve simultaneous linear equations:

$$\sum_{p=1}^{6} k_{qp} A_p + k_{qp+6} \bar{A}_p = 0, \qquad \text{for } (q = 1, 2, ..., 12), \tag{49}$$

where the values of k_{ij} , *for* $i, j = (1, 2, 3, ..., 12)$ are given in Appendix C.

The system of equations (49) has a non-trivial solution if the determinant of amplitudes $A_p, \bar{A}_p$

(p=1, 2, 3, 4, 5, 6) vanishes which leads to the secular equation

$$\left|k_{ij}\right|_{12\times 12} = 0 \qquad for\; i, j = (1,2,3,.......,12) \tag{50}$$

Equation (50) is the dispersion equation for the propagation of Stoneley waves at an interface between microstretch thermoelastic diffusion solid half spaces. This equation has complete information about the phase velocity, wave number, and attenuation coefficient of the surface waves propagating in such a medium.

7 PARTICULAR CASES

(i) In the absence of diffusion effect, the dispersion equation for the propagation of Stoneley waves at an interface between microstretch thermoelastic solid half spaces is obtained as

$$\left|k_{ij}\right|_{10\times 10} = 0 \qquad for\; i, j = (1,2,3,.......,10) \tag{51}$$

with the values of k_{ij} as

$$k_{1p} = \begin{cases} m_p^2 - b_1\xi^2 + a_2 n_{1p} + n_{2p}\left(i\xi c\tau_1 - 1\right), & for(p=1,2,3) \\ i\xi m_p\left(b_1 - 1\right), & for(p=4,5) \\ \bar{m}_{p-5}^2 - \bar{b}_1\xi^2 + \bar{a}_2\bar{n}_{1,p-5} + \bar{n}_{2,p-5}\left(i\xi c\bar{\tau}_1 - 1\right), & for(p=6,7,8) \\ i\xi\bar{m}_{p-5}\left(1-\bar{b}_1\right), & for(p=9,10) \end{cases},$$

$$k_{2p} = \begin{cases} -i\xi m_p\left(b_2 + b_3\right), & for(p=1,2,3) \\ -\left(b_2 m_p^2 + b_3\xi^2\right), & for(p=4,5) \\ i\xi\bar{m}_{p-5}\left(\bar{b}_2 + \bar{b}_3\right), & for(p=6,7,8) \\ -\left(\bar{b}_2\bar{m}_{p-5}^2 + \bar{b}_3\xi^2\right), & for(p=9,10) \end{cases}, \quad k_{3p} = \begin{cases} m_p, & for(p=1,2,3) \\ -i\xi, & for(p=4,5) \\ \bar{m}_{p-5}, & for(p=6,7,8) \\ i\xi, & for(p=9,10) \end{cases},$$

$$k_{4p} = \begin{cases} i\xi, & for(p=1,2,3) \\ m_p, & for(p=4,5) \\ -i\xi, & for(p=6,7,8) \\ \bar{m}_{p-5}, & for(p=9,10) \end{cases}, \quad k_{5p} = \begin{cases} i\xi b_5 n_{1p}, & for(p=1,2,3) \\ -b_4 n_{3p} m_p, & for(p=4,5) \\ -i\xi\bar{b}_5\bar{n}_{1,p-5}, & for(p=6,7,8) \\ -\bar{b}_4\bar{n}_{3,p-5}\bar{m}_{p-5}, & for(p=9,10) \end{cases},$$

$$k_{6p} = \begin{cases} b_6 m_p n_{1p}, & for(p=1,2,3) \\ i\xi b_5 n_{3p}, & for(p=4,5) \\ \bar{b}_6\bar{m}_{p-5}\bar{n}_{1,p-5}, & for(p=6,7,8) \\ -i\xi\bar{b}_5\bar{n}_{3,p-5}, & for(p=9,10) \end{cases}, \quad k_{7p} = \begin{cases} 0, & for(p=1,2,3) \\ n_{3p}, & for(p=4,5) \\ 0, & for(p=6,7,8) \\ -\bar{n}_{3,p-5}, & for(p=9,10) \end{cases},$$

$$k_{8p}=\begin{cases} n_{1p}, & for(p=1,2,3) \\ 0, & for(p=4,5) \\ -\bar{n}_{1,p-5}, & for(p=6,7,8) \\ 0, & for(p=9,10) \end{cases},\ k_{9p}=\begin{cases} n_{2p}, & for(p=1,2,3) \\ 0, & for(p=4,5) \\ -\bar{n}_{2,p-5}, & for(p=6,7,8) \\ 0, & for(p=9,10) \end{cases},$$

$$k_{10p}=\begin{cases} K^* m_p n_{2p}, & for(p=1,2,3) \\ 0, & for(p=4,5) \\ \bar{K}^*\bar{m}_{p-5}\bar{n}_{2,p-5}, & for(p=6,7,8) \\ 0, & for(p=9,10) \end{cases},$$

(ii) In the absence of thermal and diffusion effects, the dispersion equation (50) reduced to the propagation of Stoneley waves at an interface between microstretch elastic solid half spaces. The resulting dispersion equation in reduced form is similar as obtained by Tomer and Singh (2006).

(iii) Take $\tau^0>0,\ \varepsilon=0$ and $\gamma_1=\tau^0$ in equation (50), yield the expression of secular equation for the propagation of Stoneley waves at an interface between microstretch thermoelastic diffusion solid half spaces with two relaxation times.

(iv) Using $\tau_1=\tau^1=0, \gamma_1=\tau_0$ and $\varepsilon=1$ in equations (50), gives the corresponding results for the propagation of Stoneley waves at an interface between microstretch thermoelastic diffusion solid half spaces with with one relaxation time.

(v) On taking $\tau_0=\tau^0=\tau_1=\tau^1=\gamma_1=0$ in equations (50), provide the corresponding expression of secular equation for the propagation of Stoneley waves at an interface between microstretch thermoelastic diffusion solid half spaces with Coupled Thermoelastic (CT) theory.

8 NUMERICAL RESULTS AND DISCUSSION

The analysis is conducted for a magnesium crystal-like material. Following Eringen (1984), the values of micropolar parameters for medium M_1 are given by

$$\lambda=9.4\times10^{10}\mathrm{Nm}^{-2}, \mu=4.0\times10^{10}\mathrm{Nm}^{-2},\ K=1.0\times10^{10}\,Nm^{-2},$$
$$\rho=1.74\times10^{3}\,Kgm^{-3},\ j=0.2\times10^{-19}\,m^{2}, \gamma=0.779\times10^{-9}\,N$$

Thermal and diffusion parameters are given by

$$C^*=1.04\times10^{3}\,JKg^{-1}K^{-1},\ \mathrm{K}^*=1.7\times10^{6}\,Jm^{-1}s^{-1}K^{-1}, \alpha_{t1}=2.33\times10^{-5}\mathrm{K}^{-1},$$
$$\alpha_{t2}=2.48\times10^{-5}\mathrm{K}^{-1}, T_0=.298\times10^{3}\mathrm{K}, \tau_1=0.01, \tau_0=0.02, \alpha_{c1}=2.65\times10^{-4}\mathrm{m}^3\mathrm{Kg}^{-1},$$
$$\alpha_{c2}=2.83\times10^{-4}\mathrm{m}^3\mathrm{Kg}^{-1}, a=2.9\times10^{4}\mathrm{m}^2\mathrm{s}^{-2}\mathrm{K}^{-1}, b=32\times10^{5}\mathrm{Kg}^{-1}\mathrm{m}^5\mathrm{s}^{-2},$$
$$\tau^1=0.04, \eta^*=1.5, \tau^0=0.03, D=0.85\times10^{-8}\,Kgm^{-3}s$$

and, the microstretch parameters are taken as

$$j_o = 0.19\times10^{-19} m^2, \alpha_o = 0.779\times10^{-9} N, b_o = 0.5\times10^{-9} N, \lambda_o = 0.5\times10^{10} Nm^{-2},$$
$$\lambda_1 = 0.5\times10^{10} Nm^{-2}$$

and for medium M_2 are given by

$$\bar{\lambda} = 0.759\times10^{10}\mathrm{Nm^{-2}}, \bar{\mu} = 0.189\times10^{10}\mathrm{Nm^{-2}},\ \bar{K} = 1.49\times10^{10} Nm^{-2},$$
$$\bar{\rho} = 2.190\times10^{3} Kgm^{-3}, \bar{j} = 0.196\times10^{-19} m^2, \bar{\gamma} = 0.268\times10^{-9} N$$

Thermal and diffusion parameters are given by

$$\bar{C}^* = 1.18\times10^3 JKg^{-1}K^{-1},\ \bar{\mathrm{K}}^* = 1.5\times10^6 Jm^{-1}s^{-1}K^{-1}, \bar{\alpha}_{\mathrm{t1}} = 2.22\times10^{-5}\mathrm{K^{-1}},$$
$$\bar{\alpha}_{\mathrm{t2}} = 2.38\times10^{-5}\mathrm{K^{-1}}, \bar{T}_0 = .198\times10^3\mathrm{K}, \bar{\tau}_1 = 0.009, \bar{\tau}_0 = 0.01, \bar{\alpha}_{\mathrm{c1}} = 2.34\times10^{-4}\mathrm{m^3Kg^{-1}},$$
$$\bar{\alpha}_{\mathrm{c2}} = 2.61\times10^{-4}\mathrm{m^3Kg^{-1}}, \bar{a} = 2.32\times10^4\mathrm{m^2s^{-2}K^{-1}}, \bar{b} = 30.61\times10^5\mathrm{Kg^{-1}m^5s^{-2}},$$
$$\bar{\tau}^1 = 0.03,\ \bar{\eta}^* = 1.48, \bar{\tau}^0 = 0.02, \bar{D} = 0.63\times10^{-8} Kgm^{-3}s$$

and, the microstretch parameters are taken as

$$\bar{j}_o = 0.165\times10^{-19} m^2, \bar{\alpha}_o = 0.61\times10^{-9} N, \bar{b}_o = 0.25\times10^{-9} N, \bar{\lambda}_o = 0.37\times10^{10} Nm^{-2},$$
$$\bar{\lambda}_1 = 0.37\times10^{10} Nm^{-2}$$

MATLAB software 7.04 has been used for numerical computation of the resulting quantities. The values of phase velocity and attenuation coefficient with wave number at the stress free boundary with thermally insulated and impermeable boundaries along with the relaxation times are shown in fig.1 and fig.2. Components of displacements, normal stress, tangential couple stress, microstress, and temperature change with wave number has been determined at the surface $x_3 = 1$, and is shown in figs.3-9 for medium M_1 and in figs.10-16 for medium M_2, respectively. In all figures, for the thermally insulated boundary and impermeable boundary, the words LSWD and GLWD symbolize the graphs for L-S and G-L theories in microstretch thermoelastic diffusion medium, respectively and the words LSWTD and GLWTD symbolize the graphs for L-S and G-L theories in microstretch thermoelastic medium, respectively.

8.1 Phase Velocity

Figure 1 depicts the variation of phase velocity with frequency. In the presence of diffusion effect, the values of phase velocity for LSWD decrease monotonically and a significant difference in the values is noticed when compared in absence of diffusion LSWTD, for smaller values of frequency. The trend of variation and behavior of GLWD and GLWTD is similar to LSWD and LSWTD, respectively and for higher values of frequency, the values of phase velocity become dispersionless.

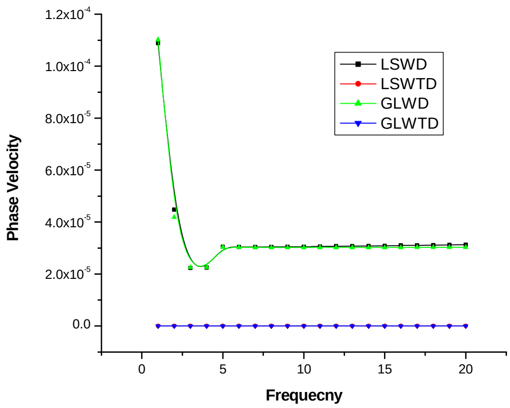


Figure1: Variation of phase velocity w.r.t frequency.

8.2 Attenuation Coefficient

Figure 2 shows that the values of attenuation coefficient increase linearly for whole range of frequency for LSWD, GLWD, LSWTD and GLWTD respectively; nevertheless, a significant difference in the values of attenuation coefficient is noticed for LSWTD, GLWTD when compared to LSWD, GLWD respectively, for whole range of frequency.

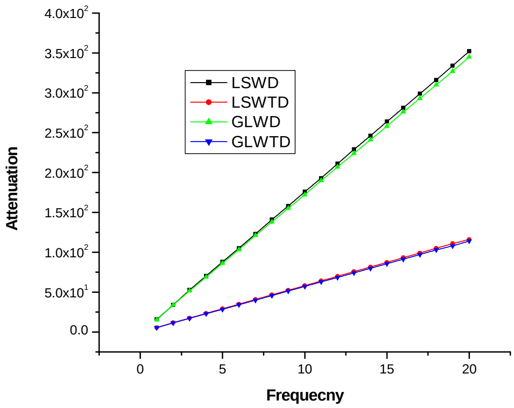


Figure 2: Variation of attenuation w.r.t frequency.

Figure3 depicts the variation of component of vertical displacement with wave number. It is noticed that, the value of u_3 decrease monotonically and oscillates afterward for smaller values of wave number in case of LSWD and GLWD, respectively, which finally become stationary for higher values of ξ. An incredible effect in LSWTD is noticed when compared to LSWD for the initial values of wave number and similar trend of variation is also noticed for GLWTD when compared to GLWD, respectively.

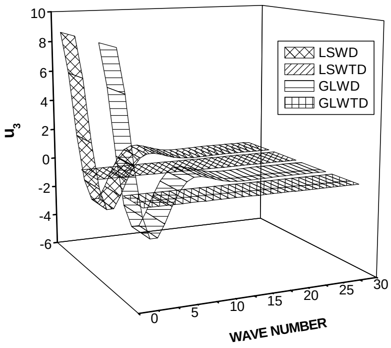
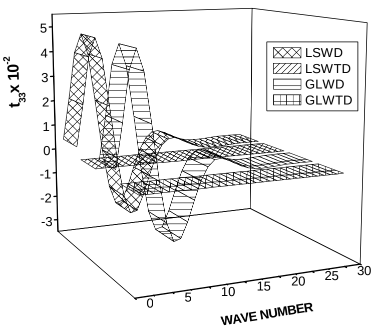


Figure 3: Variation of u_3 w.r.t wave number.

Figures 4-5 shows the trend of variation and behavior of the component of normal stress and couple stress, respectively. It is clear from the figures that due to the absence of diffusion effect, T_{33} and m_{32} remains oscillatory and become linear due to the presence of diffusion effect, for smaller values of wave number. The oscillation of T_{33} is opposite to that of m_{32}, in case of T_{33} the values first increases sharply whereas the values of m_{32} decrease monotonically for $0 \le \xi \le 5$. In Figures 4-5, LSWTD and GLWTD show a notable diffusion effect when compared to LSWD and GLWD for $16 \le \xi \le 30$, respectively.



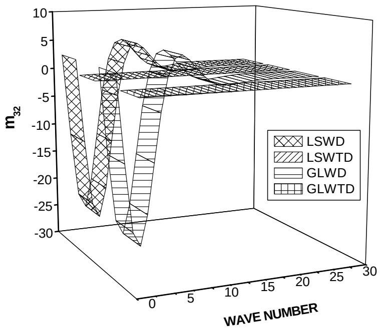


Figure 4: Variation of T_{33} w.r.t wave number.

Figure 5: Variation of m_{32} w.r.t wave number.

Figures 6-8 show the variation of λ_3^*, T and φ^* with respect to wave number, respectively. In the presence of diffusion effect, the values of λ_3^* and T become consistent for $0 \le \xi \le 30$, the similar behavior and variation is followed by φ^* with difference in the magnitude values. Nevertheless, in the absence of diffusion effect the trend of variation and behavior of λ_3^*, T and φ^* is similar, but it is oscillatory for $0 \le \xi \le 7$ and attains consistency for other values of ξ. A significant effect of diffusion is noticed for LSWTD and GLWTD when compared to LSWD and GLWD, respectively. The value of φ_2 with wave number ξ is shown in Figure9. It is clear from figure that initially there is sudden increase in values of φ_2 for LSWTD and GLWTD which become stable with the increase of ξ, in the absence of diffusion. In contrast, the values for LSWD and GLWD shows significant effect due to presence of diffusion for smaller values of ξ which shows similar trend of variation for $6 \le \xi \le 30$.

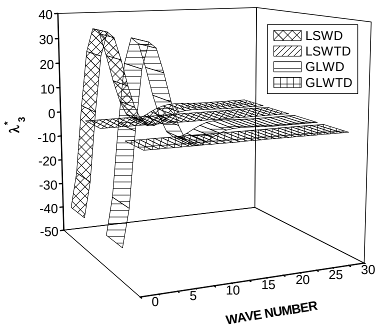


Figure 6: Variation of λ_3^* w.r.t wave number.

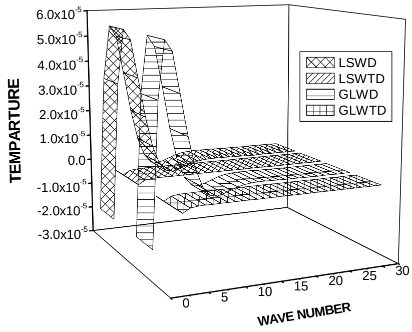


Figure 7: Variation of temperature change 'T' w.r.t wave number.

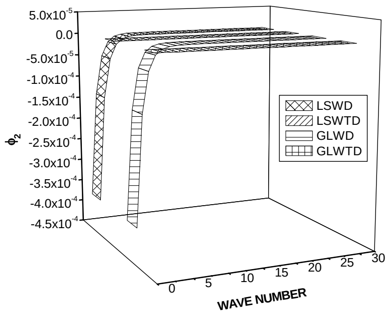


Figure 9: Variation of φ_2 w.r.t wave number.

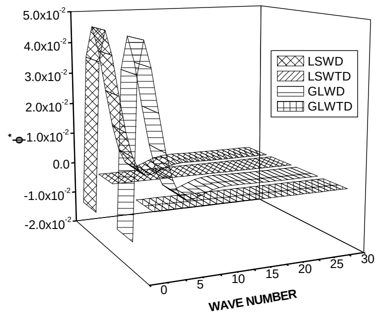
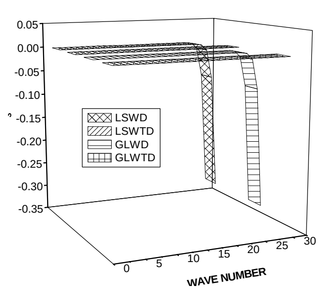


Figure 8: Variation of φ^* w.r.t wave number.

Figures10-16 shows the graph for the resulting quantities of medium M_2.



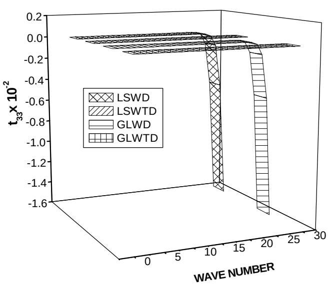


Figure 11: Variation of T_{33} w.r.t wave number.

Figure 10: Variation of u_3 w.r.t wave number.

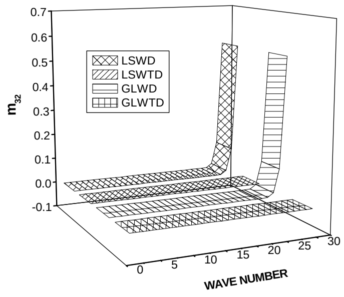


Figure 12: Variation of m_{32} w.r.t wave number.

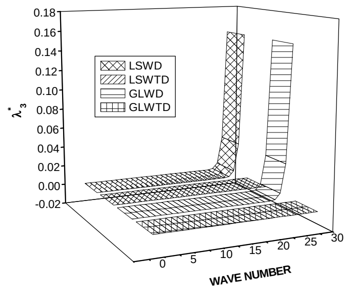


Figure 13: Variation of λ_3^* w.r.t wave number.

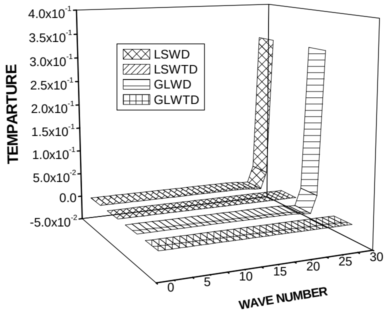


Figure 14: Variation of temperature change 'T' w.r.t wave number.

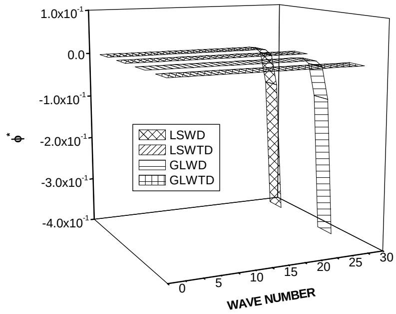


Figure 15: Variation of φ^* w.r.t wave number.

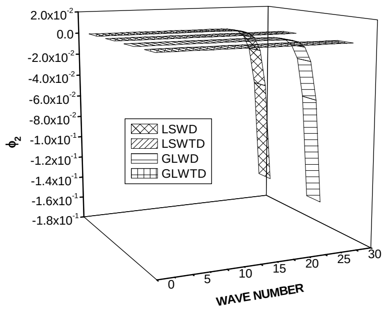


Figure 16: Variation of φ_2 w.r.t wave number.

Figure 10 depicts the variation of the component of vertical displacement with wave number. It is noticed that the trend of variation and behavior for LSWD, GLWD, LSWTD and GLWTD is similar for $0 \le \xi \le 18$ with difference in their magnitude values, but for $18 \le \xi \le 30$ the values of LSWD and GLWD decrease sharply due to presence of the diffusion effect. The similar behavior of curves is also noticed for the component of normal stress as revealed in Figure11. A significance difference in the corresponding values for LSWD and GLWD in comparison to LSWTD and

GLWED is noticed for higher values of ξ. Figures12-14 shows the variation of m_{32}, λ_3^* and T with wave number ξ, respectively. The values of m_{32} and λ_3^* for LSWD, GLWD, LSWTD and GLWTD shows a steadily linear behavior for $0 \le \xi \le 25$, but due to the existence of diffusion effect the values for LSWD and GLWD increase monotonically for other values of ξ. The effect of diffusion for LSWD and GLWD in Figures12-14 is opposite to that in Figures 10-11 which sharply decreases for $26 \le \xi \le 30$. With the gradual increase in the value of wave number for $0 \le \xi \le 22$, the values of φ^* and φ_2 shows a constant behavior for LSWD, GLWD, LSWTD and GLWTD, respectively, with the corresponding change in magnitude values, however, the values of φ^* and φ_2 for LSWD and GLWD decrease rapidly which reveals the diffusion effect with the corresponding change in magnitude values for the higher values of wave number ξ as shown in Figures15-16.

9 CONCLUSION

The propagation of Stoneley waves at the interface of two dissimilar isotropic microstretch thermoelastic diffusion medium has been investigated in the context of generalized theories of thermoelasticity. Dispersion equations of Stoneley waves for surface wave propagation are derived for the considered mathematical model. Numerical computations are performed for a particular model to study the variation of phase velocity and attenuation coefficient with respect to wave number. The components of displacement, stress, couple stress, microstress, and temperature change are computed numerically and shown graphically to depict the effect of diffusion for Lord-Shulman (1967) and Green-Lindsay (1972) theories of thermoelasticity.

It is concluded that the effect of diffusion for all the resulting quantities in medium M_1 is significant for smaller values of wave number, on the other hand, the effect of diffusion in medium M_2 is more for the higher values of wave number. Furthermore, the resulting quantities for LSWD, GLWD, LSWTD and GLWTD try to converge towards zero in medium M_1, but the same quantities attempt to diverge in medium M_2. Notable impact due to relaxation times is also revealed due to the presence of diffusion effect for every resulting quantity in both the mediums M_1 and M_2, respectively.

References

Ahmed, S. M., Abo-Dahab, S. M., (2012). Influence of initial stress and gravity field on propagation of rayleigh and stoneley waves in a thermoelastic orthotropic granular medium. Mathematical Problems in Engineering. Article ID 245965, 22 pages doi:10.1155/2012/245965.

Eringen, A.C., (1966a). Linear theory of micropolar elasticity. J. Math. Mech. 15:909–923.

Eringen, A.C., (1966b). Mechanics of micromorphic materials. Proceedings of the II International Congress of Applied Mechanics. H. Gortler (ed.), Springer, Berlin 131–138.

Eringen, A.C., (1968).Mechanics of micromorphic continua. Mechanics of Generalized Continua. E. Kroner (ed.), IUTAM Symposium, Freudenstadt-Stuttgart, Springer, Berlin 18–35.

Eringen, A.C., (1971). Micropolar elastic solids with stretch, Prof Dr Mustafa Inan Anisina, Ari Kitabevi Matbaasi, Istanbul 1–18.

Eringen, A.C., (1984). Plane wave in nonlocal micropolar elasticity. Int. J. Eng. Sci. 22:1113-1121.

Eringen, A.C., (1990). Theory of thermo-microstretch elastic solids. Int. J. Eng. Sci. 28:1291–1301.

Eringen, A.C., (1999). Microcontinuum Field Theories I: Foundations and Solids, Springer-Verlag, New York.

Iesan, D., Pompei, A., (1995).On the equilibrium theory of microstretch elastic solids. Int. J. Eng. Sci. 33:399-410.

Kumar, R., Partap, G., (2009).Wave propagation in microstretch thermoelastic plate bordered with layers of inviscid liquid. Multidiscipline Modeling in Mat. Str. 5:171-184.

Kumar, R., Deswal, S., (2002).Wave propagation through cylindrical bore contained in a microstretch elastic medium. J. Sound Vibr. 250:711-722.

Kumar, R., Garg, S.K., Ahuja, S., (2013). Propagation of plane waves at the interface of an elastic solid half-space and a microstretch thermoelastic diffusion solid half-space, Latin Journal of Solid and Structure 10:1081-1108.

Kumar, R., Kansal, T., (2008).Propagation of Lamb waves in transversely isotropic thermoelastic diffusive plate. Int. J. Sol. Struc. 45:5890-5913.

Kumar, R., Kansal, T.,(2011). Fundamental solution in the theory of thermomicrostretch elastic diffusive solids, International Scholarly Research Network, Article ID 764632, 15 pages doi:10.5402/2011/764632.

Kumar, R., Chawla, V., (2009).Wave propagation at the imperfect boundary between transversely isotropic themoelastic diffusive half space and an isotropic elastic layer.J. Tech. Phys. 50:121-146.

Tomar, S. K., Singh, D., (2006). Propagation of stoneley waves at an interface between two microstretch elastic half-spaces. Journal of Vibration and Control 12: 995–1009.

Markov, M.G. (2009). Low-frequency Stoneley wave propagation at the interface of two porous half-spaces. Geophys, J. Int. 177:603–608.

Nowacki, W., (1974a).Dynamical problems of thermodiffusion in solids-I, Bulletin of Polish Academy of Sciences Series. Science and Technology. 22: 55- 64.

Nowacki, W., (1974b).Dynamical problems of thermodiffusion in solids-II, Bulletin of Polish Academy of Sciences Series. Science and Technology 22:129-135.

Nowacki, W., (1974c).Dynamical problems of thermodiffusion in solids-III, Bulletin of Polish Academy of Sciences Series. Science and Technology 22:275-276.

Nowacki, W., (1976).Dynamical problems of diffusion in solids. Eng. Fract. Mech. 8:261-266.

Rayleigh, L., (1885). On waves propagated along the plane surface of an elastic solid. Proceedings of the London Mathematical Society 17:4–11.

Sharma,J. N.,(2007).Generalized thermoelastic diffusive waves in heat conducting materials. J. Sound Vib. 301: 979– 993.

Sharma, J. N., Sharma, Y. D., Sharma, P. K., (2008). On the propagation of elastothermodiffusive surface waves in heat conducting materials. J. Sound Vib. 315:927–938.

Sherief, H.H., Hamza, F.A., Saleh, H.A.(2004).The theory of generalized thermoelastic diffusion. Int. J.Eng. Sci. 42:591-608.

Stoneley, R.,(1924).Elastic waves at the surface of separation of two solids.Proceedings of Royal Society of London 106:416–428.

Tajuddin, M., (1995). Existence of Stoneley waves at an unbonded interface between two micropolar elastic half spaces. Journal of Applied Mechanics 62:255–257.