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Methodology for analysis of the influence of the mechanical errors and of the clearances on the precision of Watt and Stephenson mechanisms

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Abstract

This paper present a computing method used in the analysis of the precision of the six links Watt and Stephenson mechanisms. It is studied the influence of the manufacturing errors, which appear at the manufacturing of the kinematics links, on the motion law of the output link. The mathematic model is developed on the basis of the partial derivatives of the output parameters with respect to the manufacturing parameter of the mechanisms. These partial derivatives are computed with the help of the input-output equation developed for Watt and Stephenson mechanisms. The second problem presented in the paper is represented by the mathematic modeling of the precision due to the clearances from the kinematics joints. The mathematic model is developed on the basis of the Monte-Carlo method. The numeric examples lead to the conclusion regarding the choice of the tolerance interval in such a way that the precision realized by the mechanism fits in the tolerance imposed by the design.

Keywords: precision, Watt and Stephenson mechanisms

1 Introduction

The performances of the mechanisms, generally, are influenced by the manufacturing errors and by the clearances from the kinematics joints. For the choice of the tolerance and the limit deviations, for the linear and angular dimensions, the design applies the system of standards. Also this system is used in the establishment of adjustments which determine the value of the radial clearances from kinematics joints.

A first important aspect is the validation of the precision for the mechanism's motion law, on the basis of the choice of the limit deviations using the standards system, for kinematics links [5].

The researches realized until now lead to the development of two types of mathematic models for the analysis of the influence of the manufacturing errors on the motion law. The first model is analytic [4]. With the help of this model there were analyzed the influences of the manufacturing

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errors on the motion law, with numeric examples for four links mechanisms. The second model is represented by the stochastic model, which is applied generally in the precision of the mechanisms manufactured in series and of the mechanisms which are acted by random forces [3].

In the case of the six links Watt and Stephenson mechanisms the analysis of the derivations influence on the motion law is studied on each loop, with five or four links. The passing from one loop to another introduces one extra parameter (parameter which represents the position of the input link in the loop).

The clearances have important influence in the running of mechanisms and especially in the running of mechanisms close to the blocking zones. In this case it is imposed the study of the deviations of the transmission angle due to the clearances from the kinematics joints. A practical example in this way is the design of the frontal loaders.

In this paper it is presented a method for the analysis of the precision of the Watt and Stephenson mechanisms on the basis of the study of the influence of the manufacturing errors and of the clearances from the joints, using the input-output equation.

2 The input-output equation

It is determined for the Watt and Stephenson mechanisms presented in the "Fig. 1 and 2", using the method presented by [1].

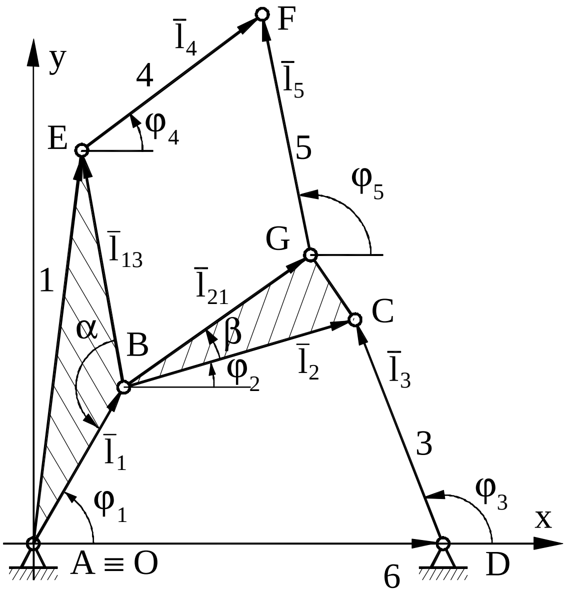

Figure 1: Watt mechanism

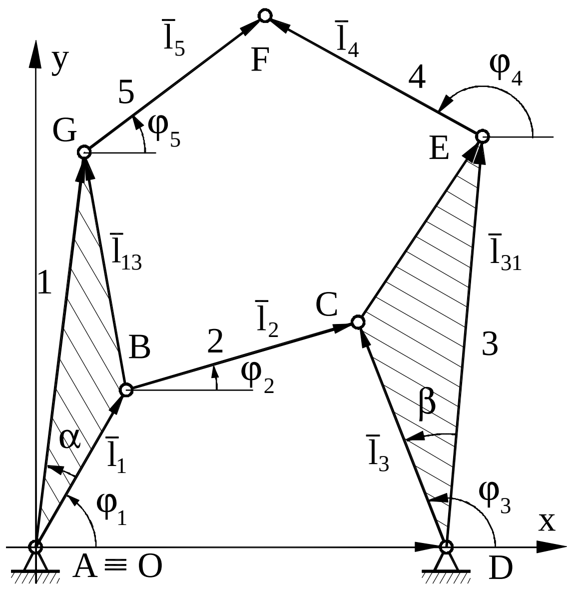

Figure 2: Stephenson mechanism

The kinematics links of the two mechanisms were numbered with number from one to six.

For both mechanisms were used the following notations: 1 – input link (AB), 5 – output link (GF), 6 – base (AD). The other links were numbered with: 2 (BC), 3 (CD), 4 (EF), 5 (FG) "Fig. 1 and 2".

The links were considered oriented vectors defined by modulus (l_1, l_2, l_3, l_6, l_{21}, l_{13}, l_4, l_5) and angular position (φ_1, φ_2, φ_3, φ_4, φ_5) regarding to the coordinate system Oxy. The angles α and β are constructive parameters "Fig. 1 and 2".

2.1 The establishment of the input-output equation for Watt mechanism

For the loop ABCDA "Fig. 1" the input- output equation is:

$$A \cdot \sin\varphi_2 + B \cdot \cos\varphi_2 + C = 0, \tag{1}$$

where:

$$A = 2 \cdot l_1 \cdot l_2 \cdot \sin\varphi_1;$$

$$B = 2 \cdot l_1 \cdot l_2 \cdot \cos\varphi_1 - 2 \cdot l_2 \cdot l_6;$$

$$C = l_1^2 + l_2^2 + l_6^2 - l_3^2 - 2 \cdot l_1 \cdot l_6 \cdot \cos\varphi_1.$$

The input-output equation for the loop BEFGB is:

$$A_1 \cdot \sin\varphi_2 + B_1 \cdot \cos\varphi_2 + C_1 = 0, \tag{2}$$

where:

$$A_1 = 2 \cdot l_{13} \cdot l_{21} \cdot \cos(\varphi_1 - \alpha) \cdot \sin\beta - 2 \cdot l_{21} \cdot l_5 \cdot \cos\varphi_5 \cdot \sin\beta + \\ + 2 \cdot l_{13} \cdot l_{21} \cdot \sin(\varphi_1 - \alpha) \cdot \cos\beta + 2 \cdot l_{21} \cdot l_5 \cdot \sin\varphi_5 \cdot \cos\beta$$

$$B_1 = 2 \cdot l_{13} \cdot l_{21} \cdot \cos(\varphi_1 - \alpha) \cdot \cos\beta + 2 \cdot l_{21} \cdot l_5 \cdot \cos\varphi_5 \cdot \cos\beta + \\ + 2 \cdot l_{13} \cdot l_{21} \cdot \sin(\varphi_1 - \alpha) \cdot \sin\beta + 2 \cdot l_{21} \cdot l_5 \cdot \sin\varphi_5 \cdot \sin\beta$$

$$C_1 = l_{13}^2 + l_{21}^2 + l_5^2 - l_4^2 + 2 \cdot l_{13} \cdot l_5 \cdot \cos(\varphi_1 - \alpha) \cdot \cos\varphi_5 + \\ + 2 \cdot l_{13} \cdot l_5 \cdot \sin(\varphi_1 - \alpha) \cdot \sin\varphi_5$$

By eliminating the angle φ_2 from the "Eq. (1)" and "Eq. (2)" and using the relation

$$\sin^2\varphi_2 + \cos^2\varphi_2 = 1,$$

results the input-output equation for the Watt mechanism presented in "Fig. 1":

$$(A_1 \cdot C - A \cdot C_1)^2 + (B \cdot C_1 - B_1 \cdot C)^2 - (A \cdot B_1 - A_1 \cdot B)^2 = 0. \tag{3}$$

2.2 The establishment of the input-output equation for Stephenson mechanism

In the case of Stephenson mechanism it is used the same method in the determination of the input-output equation. These resembles the "Eq. (3)" the difference being the structure of the computing relations for the coefficients.

For the loop ABCDA "Fig. 2" the input-output equation is:

$$D \cdot \sin\varphi_3 + E \cdot \cos\varphi_3 + F = 0,$$

where:

$$D = -2 \cdot l_1 \cdot l_3 \cdot \sin\varphi_1;$$

$$E = 2 \cdot l_3 \cdot l_6 - 2 \cdot l_1 \cdot l_3 \cdot \cos\varphi_1;$$

$$F = l_6^2 + l_3^2 + l_1^2 - l_2^2 - 2 \cdot l_1 \cdot l_6 \cdot \cos\varphi_1.$$

For input-output equation for the loop AGFEDA is:

$$D_1 \cdot \sin\varphi_3 + E_1 \cdot \cos\varphi_3 + F_1 = 0,$$

where:

$$D_1 = -2 \cdot l_{13} \cdot l_{31} \cdot \cos(\varphi_1 + \alpha) \cdot \sin\beta - 2 \cdot l_5 \cdot l_{31} \cdot \cos\varphi_5 \cdot \sin\beta - \\ -2 \cdot l_{13} \cdot l_{31} \cdot \sin(\varphi_1 + \alpha) \cdot \cos\beta - 2 \cdot l_5 \cdot l_{31} \cdot \sin\varphi_5 \cdot \cos\beta + 2 \cdot l_4 \cdot l_{31} \cdot \sin\beta\ ;$$

$$E_1 = -2 \cdot l_{13} \cdot l_{31} \cdot \cos(\varphi_1 + \alpha) \cdot \cos\beta - 2 \cdot l_5 \cdot l_{31} \cdot \cos\varphi_5 \cdot \cos\beta + \\ + 2 \cdot l_4 \cdot l_{31} \cdot \cos\beta\ + 2 \cdot l_{13} \cdot l_{31} \cdot \sin(\varphi_1 + \alpha) \cdot \sin\beta + 2 \cdot l_5 \cdot l_{31} \cdot \sin\varphi_5 \cdot \sin\varphi_3$$

$$F_1 = l_{13}^2 + l_5^2 + l_4^2 + l_{31}^2 - l_4^2 + 2 \cdot l_5 \cdot l_{13} \cdot \cos(\varphi_1 + \alpha) \cdot \cos\varphi_5 - \\ -2 \cdot l_{13} \cdot l_4 \cdot \cos(\varphi_1 + \alpha) \cdot \sin\varphi_5 - 2 \cdot l_4 \cdot l_5 \cdot \cos\varphi_5 + 2 \cdot l_{13} \cdot l_5 \cdot \sin(\varphi_1 + \alpha) \cdot \sin\varphi_5\ .$$

Repeating the computing method from subsection 2.1 it is obtained the input-output equation for the Stephenson mechanism corresponding to the "Eq. (3)":

$$(D_1 \cdot F - D \cdot F_1)^2 + (E \cdot F_1 - E_1 \cdot F)^2 - (D \cdot E_1 - D_1 \cdot E)^2 = 0.$$

As it follows it is presented the using of this equation in the study of the precision of Watt and Stephenson mechanisms.

3 The mathematic model of the precision of Watt and Stephenson mechanisms

In the process of designing the standards system imposes the limit deviations with respect to the chosen precision class. For mobile systems it is necessary to determine a mathematic model to be used to verify if the precision of the system's motion law is included in the tolerance imposed to it by the design.

The respective model must offer the design the possibility of computing the maximum and minimum values of the deviations from the given motion law, regarding to it the position of the leading link.

On the basis of the mathematical model proposed by [4] and noting with u_i, i $= 1, \ldots, n$ the input parameters and with v_j, j $= 1, \ldots, m$ the output parameters of the mechanisms, the deviations of the output parameters can be computed with the relation:

$$\{\Delta v_j\} = [J] \cdot \{\Delta u_i\} \ , \mathrm{i} = 1, \ldots, \mathrm{n} \ ; \mathrm{j} = 1, \ldots, \mathrm{m}. \tag{4}$$

where $\{\Delta v_j\}$ represents the matrix of the output parameters deviations, $\{\Delta u_i\}$ the matrix of the input parameters, and $[J]$ is the jacobian matrix:

$$[J] = \left[\frac{\partial v_j}{\partial u_i}\right] \ , \mathrm{i} = 1,...,\mathrm{n}; \ \mathrm{j} = 1,...,\mathrm{m}. \tag{5}$$

As it follows, there are applied the general "Eq. (4)" and "Eq. (5)" for the precision of the Watt and Stephenson mechanisms. So in the first phase are established the constructive parameters for the respective mechanisms:

- for Watt mechanism "Fig. 1",

$$u = u(l_1, \ l_2, \ l_3, \ l_6, \ l_{13}, \ l_{21}, \ l_4, \ l_5, \ \alpha, \ \beta \ ) \quad ; \tag{6}$$

- for Stephenson mechanism "Fig. 2",

$$u = u(l_1, \ l_2, \ l_3, \ l_{13}, \ l_{31}, \ l_4, \ l_5, \ l_6, \ \alpha, \ \beta \ ) \quad . \tag{7}$$

As output variable it is considered:

$$v \equiv \varphi_5, \tag{8}$$

where φ_5 is the parameter which define the mechanism's motion law and depends on the constructive parameters from "Eq. (6)", "Eq. (7)".

Noting with a_i the inferior limit deviation and with a_s the superior limit deviation, deviations imposed by the standards system for constructive parameters, then in the general model given by "Eq. (4)" are defined the following matrixes:

$$\{\Delta u_{ia}\} = \{a_{s1}, \ \mathrm{a}_{s2}, \ \mathrm{a}_{s3}, \ \mathrm{a}_{s6}, \ \mathrm{a}_{s13}, \ \mathrm{a}_{s21}, \ \mathrm{a}_{s4}, \ \mathrm{a}_{s5}, \ \mathrm{a}_{\mathrm{S}\alpha}, \ \mathrm{a}_{\mathrm{S}\beta}\}^T, \tag{9}$$

where $\{\Delta u_{ia}\}$ is the matrix of the superior limit deviation and

$$\{\Delta u_{ib}\} = \{a_{i1}, \ \mathrm{a}_{i2}, \ \mathrm{a}_{i3}, \ a_{i6}, \ \mathrm{a}_{i13}, \ \mathrm{a}_{i21}, \ \mathrm{a}_{i4}, \ \mathrm{a}_{i5}, \ \mathrm{a}_{i\alpha}, \ \mathrm{a}_{i\beta}\}^T \quad . \tag{10}$$

where $\{\Delta u_{ib}\}$ is the matrix of the inferior limit deviation.

These notations are used for both The Watt mechanism "Fig. 1" and Stephenson mechanism "Fig. 2".

The limit deviations, computed for the output parameter are noted with:

$$\{\Delta\, v_s\} = \{\Delta\, \varphi_{5a}\} \qquad \text{- superior deviation} \tag{11}$$

and

$$\{\Delta\, v_i\} = \{\Delta\, \varphi_{5b}\} \quad \text{- inferior deviation.} \tag{12}$$

Appling the equations "Eq. (4)", "Eq. (8)", "Eq. (9)", "Eq. (10)", "Eq. (11)" and "Eq. (12)" for the Watt mechanism, it results:

$$\{\Delta\, \varphi_{5a}\} = \left[\frac{\partial\, \varphi_5}{\partial\, \mathrm{u}_i}\right] \cdot \{\Delta\, u_{ia}\} \;\; , \mathrm{i} = 1, ..., 10, \tag{13}$$

$$\{\Delta\, \varphi_{5b}\} = \left[\frac{\partial\, \varphi_5}{\partial\, u_i}\right] \cdot \{\Delta\, u_{ib}\} \;\; , \mathrm{i} = 1,...,10\ . \tag{14}$$

The general "Eq. (13)" and "Eq. (14)" are valid for the Stephenson mechanism with the changes that appear to the constructive parameters.

The main problem is the determination of the component elements for the jacobian matrix. In this case we define the function $F\,(\varphi_5,\ u_i)$, i = 1,...,10:

$$F\left(\varphi_5,\ \mathrm{u_i}\right) = (A_1 \cdot C - A \cdot C_1)^2 + (B \cdot C_1 - B_1 \cdot C)^2 - (A \cdot B_1 - A_1 \cdot B)^2\,, \tag{15}$$

and

$$F\left(\varphi_5,\ \mathrm{u_i}\right) = 0\ , \mathrm{i} = 1, ..., 10. \tag{16}$$

Differentiating the "Eq. (16)" with respect to φ_5 and each component of the constructive parameters array it is obtained:

$$\frac{d\,\varphi_5}{d\,\mathrm{u}_i} = -\frac{\left(\frac{\partial\,\mathrm{F}}{\partial\,\mathrm{u_i}}\right)}{\left(\frac{\partial\,\mathrm{F}}{\partial\,\varphi_5}\right)}\ , \mathrm{i} = 1, ..., 10\ . \tag{17}$$

"Eq. (17)" is useful in the determining the partial derivatives of the output parameter φ_5 regarding the constructive parameters $u_i,\ i = 1, \ldots, 10$. It is remarked the simple mode of obtaining the partial derivatives, using the input - output function "Eq. (15)", " Eq. (16)". If it is considered the deviation of the input parameter, then the dimension of the array **u** increases with one, and the partial derivative in relation with this parameter is determined in the same way as above. In relation with the models determined for each loop [2], this model presents the following advantages:

- the decrease of the number of the problem's variables by eliminating the input parameter of the loop,

- simplicity of computing the partial derivatives.

For lightening the understanding of the method proposed in the paper, the following notions are defined: the tolerance realized by the mechanism, noted with T_{aj} and the imposed tolerance, noted with T. The tolerance realized by the mechanism, T_{aj}, $j = 1, ..., 360$, represents the domain in which takes values the deviation of the output parameter, when the designer imposes the standard limit deviations for the dimensions of the cinematic links. This tolerance is computed for each position of the input parameter φ_{1j} , j = 1,...,360. The tolerance T_{aj} is computed with the following equation:

$$T_{aj} = |\Delta\varphi_{5a}(\varphi_{1j}) - \Delta\varphi_{5b}(\varphi_{1j})| \, , \; j = 1, ..., 360. \tag{18}$$

If the following condition is respected:

$$\max_{j} T_{aj} \leq T \; , \tag{19}$$

then the standard limit deviations lead to the obtaining of a motion law in the limits imposed by the designer. If not, then it is applied the synthesis process and there are computed the limit deviations which respect “Eq. (19)”.

In the appendix there are presented the computing relations for the partial derivatives $\frac{\partial \mathrm{F}}{\partial \mathrm{u_i}}$, i = 1, ..., 10 and $\frac{\partial \mathrm{F}}{\partial \varphi_5}$, determined for the mechanism Watt from “Fig. 1”.

If the manufacturing errors are constants in relation with the time, the clearance from the joint is a variable parameter. The cause of this dependency is the wear phenomenon. The analysis of the clearances influence on the performances of the motion law imposes the development of a mathematical model, used to compute the tolerance realized by the mechanism.

As it follows, it is presented the development of this mathematical model and of the computing algorithm.

4 The analysis of the influence of the clearances from the joint on the motion law

The study of the influence of the clearances from the kinematics joints on the position of the output link is presented to determine the variation interval of this parameter’s deviation.

The mathematic model is obtained by raising the mobility degree of the mechanism. This situation is obtained by introducing in the mechanism’s kinematics scheme zero mass links, with the length equal to the radial clearance, named “clearance link”. The clearances space represents the existence domain of the radial clearance, noted c, and is determined by the geometry of the surfaces, which form the joint.

By example it is used the Watt mechanism, with clearances in the kinematics joints “Fig. 3”. In addition to the notations from “Fig. 1”, appear the following notations: $\overline{c}$: representing the clearance vector; O: center of the circle which forms the joint on the graphical representation of the mechanism.

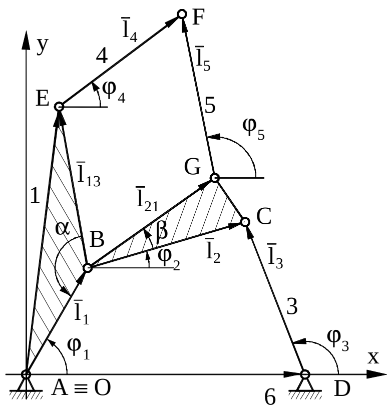

Figure 3: Watt mechanism with clearances in joints

The mechanism has eight mobility degrees: one base mobility degree, which given by the position of the input link 1; the other seven represents the positions of the clearance link in a ratio with Ox: ψ_1, ψ_2, ψ_3, ψ_4, ψ_5, ψ_6, ψ_{21} "Fig. 3". The dimensions of the clearance links are noted with: c_1, c_2, c_3, c_4, c_5, c_6, c_{21}.

The position of the output link φ_5 is computed with the relations:

$$
\begin{gathered}
M = c_1 \cdot \cos\psi_1 + c_2 \cdot \cos\psi_2 - c_6 \cdot \cos\psi_6 - c_3 \cdot \cos\psi_3; \\
N = c_1 \cdot \sin\psi_1 + c_2 \cdot \sin\psi_2 - c_6 \cdot \sin\psi_6 - c_3 \cdot \sin\psi_3; \\
A^* = 2 \cdot l_2 \cdot (l_1 \cdot \sin\varphi_1 + N); \\
B^* = 2 \cdot l_2 \cdot (l_1 \cdot \cos\varphi_1 + l_6 + M); \\
C^* = l_1^2 + l_2^2 + l_6^2 - l_3^2 + M^2 + N^2 - 2 \cdot l_6 \cdot M + \\
+ 2 \cdot l_1 \cdot M \cdot \cos\varphi_1 - 2 \cdot l_1 \cdot l_6 \cdot \cos\varphi_1 + 2 \cdot l_1 \cdot N \cdot \sin\varphi_1 ; \\
\varphi_2 = 2 \cdot a\tan\left(\frac{-A^* \pm \sqrt{A^{*2} + B^{*2} - C^{*2}}}{C^* - B^*}\right). \qquad (20) \\
P = -c_4 \cdot \cos\psi_4 + c_2 \cdot \cos\psi_2 + c_{21} \cdot \cos\psi_{21} + c_5 \cdot \cos\psi_5; \\
R = -c_4 \cdot \sin\psi_4 + c_2 \cdot \sin\psi_2 + c_{21} \cdot \sin\psi_{21} + c_5 \cdot \sin\psi_5; \\
A_3 = 2 \cdot l_{13} \cdot l_5 \cdot \sin(\varphi_1 - \alpha) + 2 \cdot l_{21} \cdot l_5 \cdot \sin(\varphi_2 + \beta) + 2 \cdot l_5 \cdot R; \\
B_3 = 2 \cdot l_{13} \cdot l_5 \cdot \cos(\varphi_1 - \alpha) + 2 \cdot l_{21} \cdot l_5 \cdot \sin(\varphi_2 + \beta) + 2 \cdot l_5 \cdot P; \\
C_3 = l_{13}^2 + l_{21}^2 + l_5^2 - l_4^2 + P^2 + R^2 + 2 \cdot l_{13} \cdot l_{21} \cdot \cos(\varphi_1 - \alpha) \cdot \cos(\varphi_2 + \beta) + \\
+ 2 \cdot l_{13} \cdot P \cdot \cos(\varphi_1 - \alpha) + 2 \cdot l_{21} \cdot P \cdot \cos(\varphi_2 + \beta) + \\
+ 2 \cdot l_{13} \cdot l_2 \cdot \sin(\varphi_1 - \alpha) \cdot \sin(\varphi_2 + \beta) + 2 \cdot l_{13} \cdot R \cdot \sin(\varphi_1 - \alpha) + 2 \cdot l_{21} \cdot R \cdot \sin(\varphi_2 + \beta); \\
\varphi_5 = 2 \cdot a\tan\left(\frac{-A_3 \pm \sqrt{A_3^2 + B_3^2 - C_3^2}}{C_3 - B_3}\right).
\end{gathered}
$$

The "Eq. (20)" was obtained with the help of the projections of the vector equations of the loops on the axes of the coordinates system "Fig. 3".

The precision of the motion law of the mechanism is analyzed using the values of the tolerance parameter T_{cj} , j $= 1, ..., 360$, computed for each position of the input link with the relation:

$$T_{cj} = \max_N \varphi_5(\varphi_{1j}) - \min_N \varphi_5(\varphi_{1j}) \text{ , j} = 1, ..., 360. \tag{21}$$

In "Eq. (21)" the main problem is the determining of the extreme values for the position φ_5, values due to the clearances from the kinematics joints.

For solving this problem it will be used the Monte-Carlo method. The computing algorithm developed on the basis of this method uses the random generation of a variable x in an interval [a,b] with the relation:

$$x = a + (b - a) \cdot R_k,\ k = 1, \ldots, N, \tag{22}$$

where: R_k are random numbers in the interval (0,1), generated using the uniform distribution.

Applying this algorithm to the presented problem and using the hypothesis of the continue contact between the links of the kinematics joints, there are generated positions of the contact's direction in the interval $[0,\ 2\pi]$, with the relation:

$$\psi_l = 2\pi \cdot R_k \text{ , k} = 1, \ldots, \text{N}, \tag{23}$$

where

$$\psi_l = \psi_l\,(\psi_1,\ \psi_2,\ \psi_3,\ \psi_6,\ \psi_{21},\ \psi_4,\ \psi_5)\,,\ l = 1, ..., 6, 21$$

and N represents the number of generations.

In this case the algorithm of the problem is:

Step 1. It is generated the vector ψ_l, with the "Eq. (23)" for each position of the input link in the interval$[0,\ 2\pi]$.

Step 2. For each position of the input link it is determined the extreme values for the angleφ_5: $\max\limits_N \varphi_5$, $\min\limits_N \varphi_5$. It is computed the tolerance T_{cj} , j $= 1, ..., 360$ with the "Eq. (21)". If $\varphi_1 = 2\pi$, go to **Step 3**.

Step 3. From the series of computed values for T_{cj}, $\max\limits_N \varphi_5$, $\min\limits_N \varphi_5$, j $= 1, ..., 360$ are established $\max\limits_j T_{cj}$, $\max\limits_j \max\limits_N \varphi_5$, $\min\limits_j \min\limits_N \varphi_5$. Corresponding to these values it is determined the positions of the clearance link ψ_l.

If it is wanted to compute the limit deviations of the mechanism's motion law, then are used the relations:

$$\max_N\ \Delta\varphi_5 = \max_N\ \varphi_5 - \varphi_5^o, \text{ for the superior deviation} \tag{24}$$

and

$$\min_N\ \Delta\varphi_5 = \min_N\ \varphi_5 - \varphi_5^o, \text{ for the inferior deviation,} \tag{25}$$

where φ_5^o represents the nominal position, which is computed with respect to the position of the input link φ_{1j} , j = 1,...,360, with the "Eq. (20)", considering *M, N, P, R* equal to zero. With this algorithm the designer can verify, for a chosen clearance, if the precision realized by the mechanism respects the tolerance imposed by the design.

The computing of the cumulate deviations. The cumulated tolerance T_{tj} , j = 1, ..., 360 realized by the mechanism is computed with the relation:

$$T_{tj} = \mathrm{T_{aj}} + T_{cj},\ j = 1, ..., 360 \tag{26}$$

where $\mathrm{T_{aj}}$ is computed with "Eq. (18)" and T_{cj} with "Eq. (21)".

In the same way there are computed the superior and inferior cumulated deviations:

$$\begin{aligned} \Delta\varphi_{5M}(\varphi_{1j}) &= \Delta\varphi_{5a}(\varphi_{1j}) + \max_N \Delta\varphi_5(\varphi_{1j}), \\ \Delta\varphi_{5m}(\varphi_{1j}) &= \Delta\varphi_{5b}(\varphi_{1j}) + \min_N \Delta\varphi_5(\varphi_{1j}),\ j = 1, ..., 360, \end{aligned} \tag{27}$$

$\Delta\varphi_{5M}-$ the superior cumulated deviation and $\Delta\varphi_{5m}$ - the inferior cumulated deviation.

In this case, "Eq. (19)" and "Eq. (21)" are transformed in:

$$\max_j T_{tj} \leq T. \tag{28}$$

If "Eq. (28)" is true, then the motion law is in the limits imposed, if not, then in the first phase it is modified the deviations or the clearances in the joints with the maximum weight from the cumulated value of the tolerance. If after that, "Eq. (28)" is not true, then there are modified both components on the basis of the synthesis process using an optimization algorithm.

5 Numerical examples. Discussions

5.1 The establishment of the domain of the variation of the motion law deviation due to the manufacturing errors

It is considered the mechanism from "Fig. 1", with the following nominal dimensions for kinematics links:

$$l_1 = 0.090 \text{ [m]},\quad l_2 = 0.210 \text{ [m]},\quad l_3 = 0.210 \text{ [m]},\quad l_6 = 0.320 \text{ [m]},$$

$$l_{13} = 0.045 \text{ [m]},\quad l_{21} = 0.150 \text{ [m]},\quad l_4 = 0.150 \text{ [m]},\quad l_5 = 0.150 \text{ [m]},$$

$$\alpha = 115^\circ,\quad \beta = 30^\circ.$$

For these dimensions, the limit deviations imposed by the standards system are:

$$a_1 = \pm\ 0.15 \text{ [mm]},\quad a_2 = \pm\ 0.20 \text{ [mm]},\quad a_3 = \pm\ 0.20 \text{ [mm]},\quad a_6 = \pm\ 0.30 \text{ [mm]},$$

$$a_{13} = \pm\ 0.15\ [\mathrm{mm}],\quad a_{21} = \pm\ 0.20\ [\mathrm{mm}],\quad a_4 = \pm\ 0.20\ [\mathrm{mm}],\quad a_5 = \pm\ 0.20\ [\mathrm{mm}],$$

$$a_\alpha = \pm\ 50\ ',\quad a_\beta = \pm\ 50\ '.$$

In the "Figures 4, 5 and 6" are represented the variations diagrams of the partial derivatives regarding to the position of the input linkφ_1.

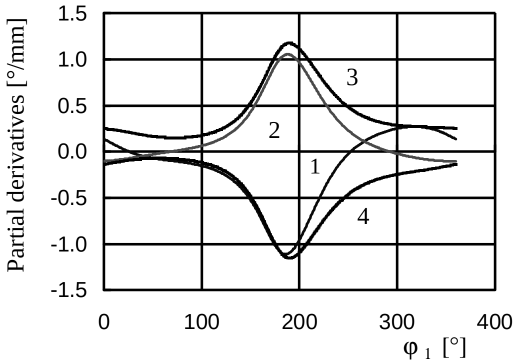

Figure 4: The variation of the partial derivatives $\frac{\partial\varphi_5}{\partial l_1}-1,\ \frac{\partial\varphi_5}{\partial l_2}-2,\ \frac{\partial\varphi_5}{\partial l_3}-3,\ \frac{\partial\varphi_5}{\partial l_6}-4$

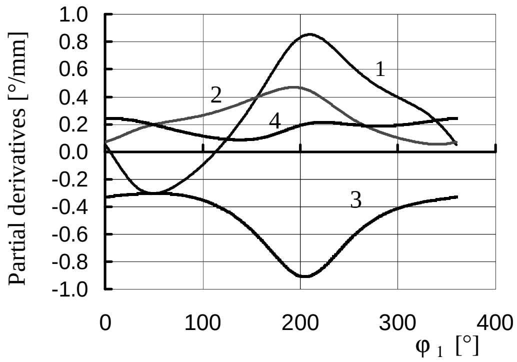

Figure 5: The variation of the partial derivatives $\frac{\partial\varphi_5}{\partial l_{13}}-1,\ \frac{\partial\varphi_5}{\partial l_{21}}-2,\ \frac{\partial\varphi_5}{\partial l_4}-3,\ \frac{\partial\varphi_5}{\partial l_5}-4$

Analyzing "Figures 4 and 5", from the point of view of the modules for the values of the partial derivatives, it is noticed a relative important influence of the partial derivatives which belong to the loop ABCDA with respect to the partial derivatives from the loop BEFGB. In "Fig. 6" can be observed an important influence of the parameter β, regarding to α and the other parameters from "Figures 4 and 5".

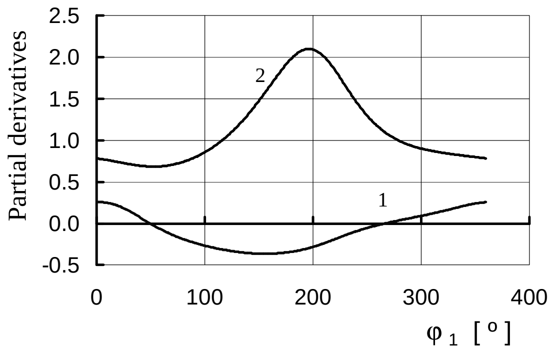

Figure 6: The variation of the partial derivatives

$$\frac{\partial\varphi_5}{\partial\alpha}-1,\ \frac{\partial\varphi_5}{\partial\beta}-2$$

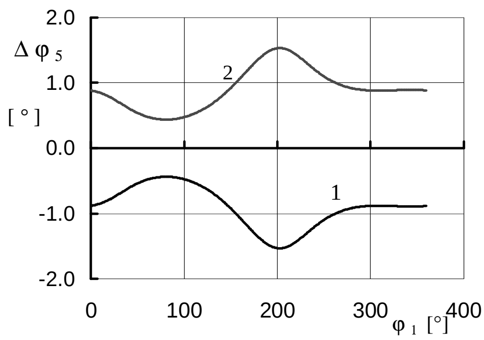

Figure 7: The domain of the variation output link deviations

$$\Delta\varphi_{5b}-1, \Delta\varphi_{5a}-2$$

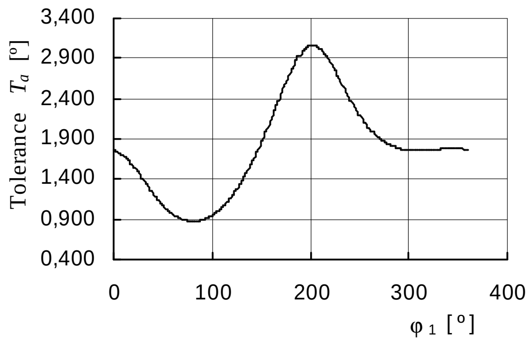

Figure 8: The domain of the variation of the tolerance T_a

It can be remarked the special influence that the angular parameters α, β and the constructive parameters of the loop BEFGB have on the mechanism's motion law. This is explained by the fact that the algebraic sum of the partial derivatives with respect to the constructive parameters of the first loop "Fig. 4" is approximately zero.

In the "Fig. 7" and "Fig. 8" there are presented the variation domains of the deviations of the output parameter $\Delta\varphi_5(\Delta\varphi_{5a}, \Delta\varphi_{5b})$ and for the tolerance T_{aj}, realized by the mechanism.

5.2 The computing of the variation of the deviations due to the clearances from the kinematics joints

The computing example is applied for the mechanism presented in "Fig. 3". There are considered the same nominal dimensions as in the previous example, and the radial clearances $c_k = 0.15$ [mm] , k = 1, 2, 3, 4, 5, 6, 21. The number of generation is N = 10000 and applying the algorithm presented at section 4, developed on the basis of the "Eq. (20)", "Eq. (21)", "Eq. (22)" and "Eq. (23)", it is obtained the diagram of the variation of the tolerance realized by the mechanism "Fig. 3", for each position of the input link.

In "Fig. 9" it is presented the variation of the tolerance $T_{cj}, j = 1, ..., 360$ regarding to the position of the input parameter. The numeric results from Table 1 show the positions of the cinematic elements of the Watt mechanism for the maximum value of the tolerance $T_{cj}, j = 1, ..., 360$. The simulation of the maximum and minimum positions of the cinematic elements leads to the design of a product that avoids the eventual blockage positions and respects the conditions imposed to the motion law.

The performance of the computing algorithm is analyzed by studying the variation of the tolerance $T_{cj}, j = 1, ..., 360$ with respect to the number of generations N "Fig. 10" and the number of the executions of the programs n_e "Fig. 11". For N = 25.000 it is observed that the solution is stabilized near the value of 0.96°.

Also, the maximum value of the tolerance T_c depends on n_e "Fig. 11". It is observed an equilibration of the solution near the value of 0.953°.

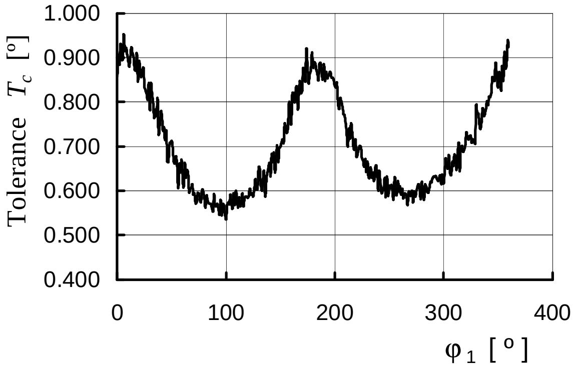

Figure 9: The domain of the variation of the tolerance T_c

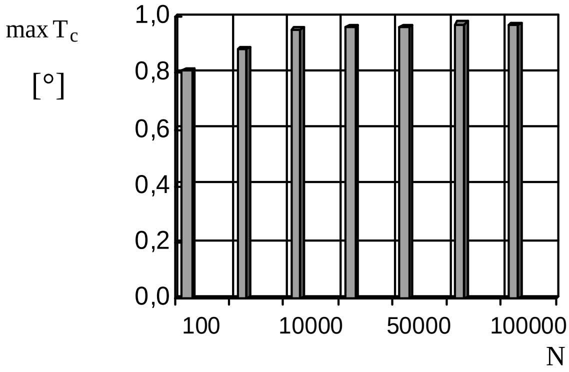

Figure 10: The value of the tolerance with respect to N

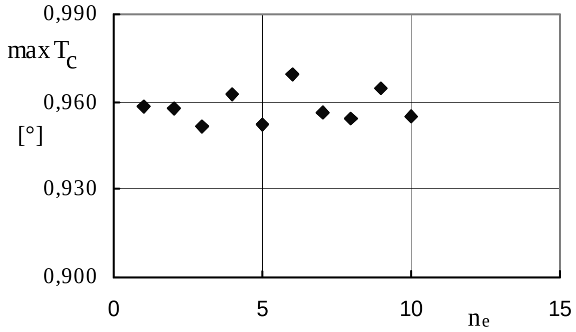

Figure 11: The value of the tolerance regarding to n_e

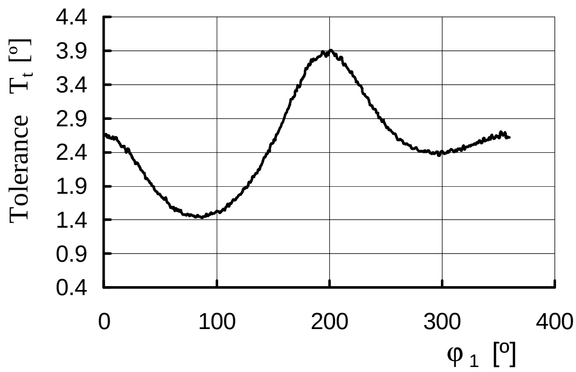

Figure 12: The domain of the variation of the tolerance T_t

Table 1: Numeric results

		φ_1 [°]	ψ_1 [°]	ψ_2 [°]	ψ_3 [°]	ψ_4 [°]	ψ_5 [°]	ψ_6 [°]	ψ_{21}[°] [°]
$\max_j T_{cj}$[°]	0.96818	185	-	-	-	-	-	-	-
$\max_j \max_N \varphi_5$[°]	173.568	185	344.75	37.07	178.13	276.40	139.46	168.34	118.14
$\min_j \min_N \varphi_5$ [°]	172.63	185	225.0	340.34	106.91	332.26	5.32	312.55	106.28

The simplicity of this method is an important advantage with respect to the application of the numerical methods because of the fact that it does not require an initial solution and that it avoids the development of a complex mathematical model.

The additive effect of the limit deviations and of the clearances from the joints is presented in "Fig. 12". The values of the additive tolerance T_t are computed with "Eq. 26". The validity of the obtained motion law is imposed by "Eq. 28".

6 Conclusion

The development of an analytic model for the study of the precision of Watt and Stephenson mechanisms, using the input-output equation, for determining the partial derivatives regarding to constructive parameters, offers simplicity and easiness in applying.

Regarding to the analysis of the precision on separates loop, in which is necessary the study of the influence of the deviation of the input position of the next loop, the analyzed method is applied for the whole mechanism using only the limit deviations of the constructive parameters, leading to the obtaining of a high relative precision. The study of the influence of the partial derivatives on the precision realized by the mechanism may lead the designer to the establishment of the tolerance intervals for each constructive parameter, in such a way that the tolerance imposed to the motion law by the design is respected.

The analysis of the influence of the deviation produced by the clearances from the kinematics joints on the motion law of Watt and Stephenson mechanisms can be easily studied using the Monte-Carlo method. This method is characterized by generality and can be used successfully in any design workshop.

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Appendix

The determination of the partial derivatives for the Watt mechanism "Fig. 1"

I. The derivate of the function $F(\varphi_5, u_i)$, i = 1,...,10 with respect to the constructive parameters u_i:

$$\frac{\partial F}{\partial u_i} = 2(A_1C - AC_1)\left(\frac{\partial A_1}{\partial u_i}C + A_1\frac{\partial C}{\partial u_i} - C_1\frac{\partial A}{\partial u_i} - A\frac{\partial C_1}{\partial u_i}\right) + 2(BC_1 - B_1C)\left(\frac{\partial B}{\partial u_i}C_1 + B\frac{\partial C_1}{\partial u_i} - C\frac{\partial B_1}{\partial u_i} - B_1\frac{\partial C}{\partial u_i}\right) - 2(AB_1 - A_1B)\left(\frac{\partial A}{\partial u_i}B_1 + A\frac{\partial B_1}{\partial u_i} - B\frac{\partial A_1}{\partial u_i} - A_1\frac{\partial B}{\partial u_i}\right)$$

II. Partial derivatives

1. **Partial derivatives with respect to parameter l_1:**

$$\frac{\partial A}{\partial l_1} = 2l_2\sin\varphi_1 \quad \frac{\partial B}{\partial l_1} = 2l_2\cos\varphi_1 \quad \frac{\partial C}{\partial l_1} = 2l_1 - 2l_6\cos\varphi_1$$

$$\frac{\partial A_1}{\partial l_1} = 0 \quad \frac{\partial B_1}{\partial l_1} = 0 \quad \frac{\partial C_1}{\partial l_1} = 0$$

2. **Partial derivatives with respect to parameter l_2:**

$$\frac{\partial A}{\partial l_2} = 2l_1\sin\varphi_1 \quad \frac{\partial B}{\partial l_2} = 2l_1\cos\varphi_1 - 2l_6 \quad \frac{\partial C}{\partial l_2} = 2l_2$$

$$\frac{\partial A_1}{\partial l_2} = 0 \quad \frac{\partial B_1}{\partial l_2} = 0 \quad \frac{\partial C_1}{\partial l_2} = 0$$

3. **Partial derivatives with respect to parameter l_3:**

$$\frac{\partial A}{\partial l_3} = 0 \quad \frac{\partial B}{\partial l_3} = 0 \quad \frac{\partial C}{\partial l_3} = -2l_3$$

$$\frac{\partial A_1}{\partial l_3} = 0 \quad \frac{\partial B_1}{\partial l_3} = 0 \quad \frac{\partial C_1}{\partial l_3} = 0$$

4. **Partial derivatives with respect to parameter l_6:**

$$\frac{\partial A}{\partial l_6} = 0 \quad \frac{\partial B}{\partial l_6} = -2l_2 \quad \frac{\partial C}{\partial l_6} = 2l_6 - 2l_1\cos\varphi_1$$

$$\frac{\partial A_1}{\partial l_6} = 0 \quad \frac{\partial B_1}{\partial l_6} = 0 \quad \frac{\partial C_1}{\partial l_6} = 0$$

5. **Partial derivatives with respect to parameter l_{13}:**

$$\frac{\partial A}{\partial l_{13}} = 0 \quad \frac{\partial B}{\partial l_{13}} = 0 \quad \frac{\partial C}{\partial l\ _{13}} = 0$$

$$\frac{\partial A_1}{\partial l_{13}} = 2l_{21}\sin(\varphi_1 - \alpha - \beta) \quad \frac{\partial B_1}{\partial l_{13}} = 2l_{21}\cos(\varphi_1 - \alpha - \beta) \quad \frac{\partial C_1}{\partial l_{13}} = 2l_{13} + 2l_5\cos(\varphi_1 - \alpha - \beta)$$

6. Partial derivatives with respect to parameter α :

$$\frac{\partial A}{\partial \alpha} = 0 \quad \frac{\partial B}{\partial \alpha} = 0 \quad \frac{\partial C}{\partial \alpha} = 0$$

$$\frac{\partial A_1}{\partial \alpha} = -2l_{13}l_{21}\cos(\varphi_1 - \alpha - \beta) \quad \frac{\partial B_1}{\partial \alpha} = 2l_{13}l_{21}\sin(\varphi_1 - \alpha - \beta) \quad \frac{\partial C_1}{\partial \alpha} = 2l_{13}l_5\sin(\varphi_1 - \alpha - \varphi_5)$$

7. Partial derivatives with respect to parameter l_4:

$$\frac{\partial A}{\partial l_4} = 0 \quad \frac{\partial B}{\partial l_4} = 0 \quad \frac{\partial C}{\partial l_4} = 0 \quad \frac{\partial A_1}{\partial l_4} = 0 \quad \frac{\partial B_1}{\partial l_4} = 0 \quad \frac{\partial C_1}{\partial l_4} = -2l_4$$

8. Partial derivatives with respect to parameter l_5:

$$\frac{\partial A}{\partial l_5} = 0 \quad \frac{\partial B}{\partial l_5} = 0 \quad \frac{\partial C}{\partial l\ _5} = 0$$

$$\frac{\partial A_1}{\partial l_5} = 2l_{21}\sin(\varphi_5 - \beta) \quad \frac{\partial B_1}{\partial l_5} = 2l_{21}\cos(\varphi_5 - \beta) \quad \frac{\partial C_1}{\partial l_5} = 2l_5 + 2l_{13}\cos(\varphi_1 - \alpha)\cos\beta$$

9. Partial derivatives with respect to parameter l_{21}:

$$\frac{\partial A}{\partial l_{21}} = 0 \quad \frac{\partial B}{\partial l_{21}} = 0 \quad \frac{\partial C}{\partial l\ _{21}} = 0$$

$$\frac{\partial A_1}{\partial l_{21}} = 2l_{13}\sin(\varphi_1 - \alpha - \beta) + 2l_5\sin(\varphi_5 - \beta) \quad \frac{\partial B_1}{\partial l_{21}} = 2l_{13}\cos(\varphi_1 - \alpha - \beta) + 2l_5\cos(\varphi_5 - \beta) \quad \frac{\partial C_1}{\partial l_{21}} = 2l_{21}$$

10. Partial derivatives with respect to parameter β:

$$\frac{\partial A}{\partial \beta} = 0 \quad \frac{\partial B}{\partial \beta} = 0 \quad \frac{\partial C}{\partial \beta} = 0$$

$$\frac{\partial A_1}{\partial \beta} = -2l_{13}l_{21}\cos(\varphi_1 - \alpha - \beta) - 2l_{21}l_5\cos(\varphi_5 - \beta)$$

$$\frac{\partial B_1}{\partial \beta} = 2l_{13}l_{21}\sin(\varphi_1 - \alpha - \beta) + 2l_{21}l_5\sin(\varphi_5 - \beta) \quad \frac{\partial C_1}{\partial \beta} = 0$$

III. The derivative of the function $F\left(\varphi_5, u_i\right)$, i = 1,...,10:

$$\frac{\partial F}{\partial \varphi_5} = 2(A_1C - AC_1)(\frac{\partial A_1}{\partial \varphi_5}C + A_1\frac{\partial C}{\partial \varphi_5} - C_1\frac{\partial A}{\partial \varphi_5} - A\frac{\partial C_1}{\partial \varphi_5}) + 2(BC_1 - B_1C)(\frac{\partial B_1}{\partial \varphi_5}C_1 + \\ + B\frac{\partial C_1}{\partial \varphi_5} - C\frac{\partial B_1}{\partial \varphi_5} - B_1\frac{\partial C}{\partial \varphi_5}) - 2(AB_1 - A_1B)(\frac{\partial A}{\partial \varphi_5}B_1 + A\frac{\partial B_1}{\partial \varphi_5} - B\frac{\partial A_1}{\partial \varphi_5} - A_1\frac{\partial B}{\partial \varphi_5})$$

1. **Partial derivatives with respect to parameter φ_5:**

$$\frac{\partial A}{\partial \varphi_5} = 0 \quad \frac{\partial B}{\partial \varphi_5} = 0 \quad \frac{\partial C}{\partial \varphi_5} = 0$$

$$\frac{\partial A_1}{\partial \varphi_5} = 2l_5l_{21}\cos(\varphi_5 - \beta) \quad \frac{\partial B_1}{\partial \varphi_5} = -2l_5l_{21}\sin(\varphi_5 - \beta) \quad \frac{\partial C_1}{\partial \varphi_5} = 2l_5l_{13}\sin(\varphi_1 - \alpha - \varphi_5)$$